

Financial Econometrics

HT Week 5 Assignment Answers

February 28, 2019

Exercise 8.5

A Value-at-Risk model was fit to some return data and the series of 5% VaR violations was computed. Denote these \widehat{HIT}_t . The total number of observations was $T = 50$, and the total number of violations was 4.

1. **Test the null that the model has unconditionally correct coverage using a t -test.**

Since there were 4 out of 50 violations, $\hat{p} = 4/50 = 8\%$. This is just the mean, so that we can estimate the variance using

$$\begin{aligned} 50^{-1} \sum_{i=1}^{50} (x_i - \bar{x})^2 &= 50^{-1} \sum_{i=1}^{50} x_i^2 - 2x_i \bar{x} + \bar{x}^2 \\ &= 50^{-1} \sum_{i=1}^{50} x_i^2 - 50\bar{x}^2 \end{aligned}$$

The components of this sum as $50^{-1} \sum_{i=1}^{50} x_i^2 = 50^{-1} \sum_{i=1}^{50} x_i = \hat{p}$ since $x \in \{0, 1\}$ and $50^{-1} (50) \bar{x}^2 = \hat{p}^2$, and so the variance estimator is $\hat{p}(1 - \hat{p})$. Finally, the variance of the mean is $(n - 1)^{-1} (p(1 - p))$ which can be estimated using $(n - 1)^{-1} \hat{p}(1 - \hat{p})$ and so

$$\frac{\hat{p} - 0.05}{\sqrt{(n - 1)^{-1} (\hat{p}(1 - \hat{p}))}} = \frac{.03}{0.0388} = 0.7741$$

The 2-sided critical value from a normal for a 5% test is ± 1.96 and so the null cannot be rejected.

2. **Test the null that the model has unconditionally correct coverage using a LR test. The likelihood for a Bernoulli(p) random Y is**

$$f(y; p) = p^y (1 - p)^{1-y}.$$

The log likelihood is

$$\sum_{i=1}^{50} y_i \ln p + (1 - y_i) \ln(1 - p)$$

and so the MLE can be found by differentiating

$$\begin{aligned}\frac{\partial}{\partial p} &= \sum_{i=1}^{50} \frac{y_i}{\hat{p}} - \frac{1-y_i}{1-\hat{p}} = 0 \\ 0 &= \sum_{i=1}^{50} (1-\hat{p})y_i - (1-y_i)\hat{p} \\ 0 &= \sum_{i=1}^{50} y_i - y_i\hat{p} - \hat{p} + y_i\hat{p} \\ n\hat{p} &= \sum_{i=1}^{50} y_i \\ \hat{p} &= \bar{y}\end{aligned}$$

Using this I can now compute the likelihood ratio against the null model where $p = 5\%$,

$$\begin{aligned}2 \times \left\{ \sum_{i=1}^{50} y_i \ln .08 + (1-y_i) \ln .92 - \sum_{j=1}^{50} y_j \ln .05 + (1-y_j) \ln .95 \right\} \\ = 2 \times \{4 \ln .08 + 46 \ln .92 - 4 \ln .05 - 46 \ln .95\} \\ = 2 \times \{4(\ln .08 - \ln .05) + 46(\ln .92 - \ln .95)\} = 0.807\end{aligned}$$

There is one restriction and so the distribution is a χ_1^2 and the critical value is 3.84, and so we fail to reject the null.

The following regression was estimated

$$\widetilde{HIT}_t = 0.0205 + 0.7081\widetilde{HIT}_{t-1} + \hat{\eta}_t$$

The estimated asymptotic covariance of the parameters is

$$\hat{\sigma}^2 \hat{\Sigma}_{XX}^{-1} = \begin{bmatrix} 0.0350 & -0.0350 \\ -0.0350 & 0.5001 \end{bmatrix}, \text{ and } \hat{\Sigma}_{XX}^{-1} \hat{\Sigma}_{XX}^{-1} = \begin{bmatrix} 0.0216 & -0.0216 \\ -0.0216 & 2.8466 \end{bmatrix}$$

where $\hat{\sigma}^2 = \frac{1}{T} \sum_{t=1}^T \hat{\eta}_t^2$, $\hat{\Sigma}_{XX} = \frac{1}{T} \mathbf{X}'\mathbf{X}$ and $\hat{\mathbf{S}} = \frac{1}{T} \sum_{t=1}^T \hat{\eta}_t^2 \mathbf{x}_t' \mathbf{x}_t$.

3. Is there evidence that the model is dynamically mis-specified, ignoring the unconditional rate of violations?

This can be tested using a basis t-test on the lagged term. Since the data are Bernoulli, I need to use the heteroskedasticity robust standard errors. The null is that the coefficient is 0.

$$\frac{.7081 - 0}{\sqrt{2.8466/50}} = 2.96$$

Using a 5% test, the critical value is ± 1.96 and so the null is rejected.

4. Compute a joint test that the model is completely correctly specified. Note that

$$\begin{bmatrix} a & b \\ b & c \end{bmatrix}^{-1} = \frac{1}{ac - b^2} \begin{bmatrix} c & -b \\ -b & a \end{bmatrix}.$$

We can use a Wald test of the form

$$T(\hat{\theta} - \theta_0)' \hat{\Sigma}^{-1} (\hat{\theta} - \theta_0)$$

to test 2 restrictions, $H_0 : \theta_0 = 0.05, \hat{\theta}_1 = 0$. Using the inverse formula above,

$$\hat{\Sigma}^{-1} = \begin{bmatrix} 46.6 & 0.353 \\ 0.353 & 0.353 \end{bmatrix}$$

so that the value of the Wald statistic is 10.1648. This test has 2 restrictions, and so the critical value is 5.99. In this case, we reject.

Note: The 5% critical values of a χ^2_ν are

ν	CV
1	3.84
2	5.99
3	7.81
47	64.0
48	65.1
49	66.3
50	67.5