A13073W1

DEGREE OF MASTER OF SCIENCE IN FINANCIAL ECONOMICS

FINANCIAL ECONOMETRICS

TRINITY TERM 2018

Tuesday, 17 April 2018, 9:30 AM – 12:00 PM

Time allowed is TWO HOURS AND THIRTY MINUTES.

*Candidates should answer **ALL** questions in part A.*
*Candidates should answer **TWO** of three questions in part B.*

Examiners will place weight 6.25% on each short answer and 25% on each long answer.

*Use **three** booklets – one for Part A and one for each Part B question.*
Write the numbers for B questions answered on the cover of the relevant booklet.

Materials: Calculators.
Calculators must not be removed from the Examination Room.

Do not turn over until told that you may do so.

Part A: Short Answer

Answer ALL questions in this section.

Each question is worth 6.25% of the exam mark. (i.e. 1/8 of 50%).

1. If $(X,\epsilon) \sim N(0, I_2)$ what is the distribution of $Y = \mu + \beta X + \epsilon$?
2. What is the Cramer-Rao lower bound and why is it useful?
3. Derive the OLS estimator for the model $y_i = \beta x_i + \epsilon_i$.
4. Describe the steps to implement k-fold cross validation in a regression to select a model.
5. Under what conditions on ϕ_0, ϕ_1 and θ_1 is the process $\{y_t\}$ stationary where $y_t = \phi_0 + \phi_1 y_{t-1} + \theta_1 \epsilon_{t-1} + \epsilon_t$ and $\{\epsilon_t\}$ is a white noise process.
6. What properties must a covariance stationary time series satisfy?
7. Outline the steps the in Mincer-Zarnowitch framework to objectively evaluate a sequence of variance forecasts $\left\{\hat{\sigma}^2_{t+1|t}\right\}$.
8. What is Principal Component Analysis and how is PCA useful in covariance modeling?

Part B: Long Answer

Answer TWO of the three questions in this section.

Each question is worth 25% of the exam mark (i.e., 1/2 of 50%). Within each question points sum to 100% and so will be scaled by 25% when combined in the final exam mark.

1. Consider the APT regression

$$r_t^e = \alpha + \beta_m r_{m,t}^e + \beta_s r_{smb,t} + \beta_v r_{hml,t} + \epsilon_t$$

where $r_{m,t}^e$ is the excess return on the market, $r_{smb,t}$ is the return on the size factor, $r_{hml,t}$ is the return on value factor and r_t^e is an excess return on a portfolio of assets. Using the information provided in the tables below below, answer the following questions:

   (a) [10%] Is there evidence that this portfolio is market neutral?

   (b) [10%] Are the size and value factors needed in this portfolio to adequately capture the cross-sectional dynamics?

   (c) [20%] Is there evidence of conditional heteroskedasticity in this model?

   (d) [10%] What are the trade-offs for choosing a covariance estimator for making inference on this model?

   (e) [15%] Define the size and power of a statistical test.

   (f) [15%] What factors affect the power of a statistical test?

   (g) [20%] Outline the steps to implement the correct bootstrap covariance estimator for these parameters. Justify the method you chose using the information provided.

Notes: All models were estimated on $n = 100$ data points. Models 1 and 2 correspond to the specification above. In model 1 r_{smb} and r_{hml} have been excluded. Model 3, 4 and 5 are all version of

$$\hat{\epsilon}_t^2 = \gamma_0 + \gamma_1 r_{m,t}^e + \gamma_2 r_{smb,t} + \gamma_3 r_{hml,t} + \gamma_4 \left(r_{m,t}^e\right)^2 + \gamma_5 r_{m,t}^e r_{smb,t} + \gamma_6 r_{m,t}^e r_{hml,t} + \gamma_7 r_{smb,t}^2 + \gamma_8 r_{smb,t} r_{hml,t} + \gamma_9 r_{hml,t}^2 + \eta_t$$

$\hat{\epsilon}_t$ was computed using Model 1 for the results under Model 3, and using model 2 for the results under Models 4 and 5. R^2 is the R-squared and n is the number of observations.

Parameter Estimates

	Model 1	Model 2		Model 3	Model 4	Model 5
α	0.128	0.089	γ_0	0.984	0.957	0.931
β_m	1.123	0.852	γ_1	-0.779	-0.498	
β_{smb}		0.600	γ_2		-0.046	
β_{hml}		-0.224	γ_3		0.124	
			γ_4	0.497	0.042	0.295
			γ_5		0.049	
			γ_6		0.684	
			γ_7		0.036	-0.149
			γ_8		-0.362	
			γ_9		-0.005	0.128
R^2	0.406	0.527		0.134	0.126	0.037

Parameter Covariance Estimates

The estimated covariance matrices from the asymptotic distribution

$$\sqrt{n}\left(\hat{\boldsymbol{\beta}} - \hat{\boldsymbol{\beta}}_0\right) \xrightarrow{d} N(0, C)$$

are below where C is either $\hat{\sigma}^2\hat{\Sigma}_{XX}^{-1}$ or $\hat{\Sigma}_{XX}^{-1}\hat{S}\hat{\Sigma}_{XX}^{-1}$.

CAP-M

$\hat{\sigma}^2\hat{\Sigma}_{XX}^{-1}$

	α	β_m
α	1.365475	0.030483
β_m	0.030483	1.843262

$\hat{\Sigma}_{XX}^{-1}\hat{S}\hat{\Sigma}_{XX}^{-1}$

	α	β_m
α	1.341225	-0.695235
β_m	-0.695235	2.747142

Fama-French Model

$\hat{\sigma}^2\hat{\Sigma}_{XX}^{-1}$

	α	β_m	β_{smb}	β_{hml}
α	1.100680	0.103611	-0.088259	-0.063529
β_m	0.103611	1.982761	-0.619139	-0.341118
β_{smb}	-0.088259	-0.619139	1.417318	-0.578388
β_{hml}	-0.063529	-0.341118	-0.578388	1.686200

$\hat{\Sigma}_{XX}^{-1}\hat{S}\hat{\Sigma}_{XX}^{-1}$

	α	β_m	β_{smb}	β_{hml}
α	1.073227	-0.361618	-0.072784	0.045732
β_m	-0.361618	2.276080	-0.684809	0.187441
β_{smb}	-0.072784	-0.684809	1.544745	-1.074895
β_{hml}	0.045732	0.187441	-1.074895	1.947117

χ^2_m critical values

Critical value for a 5% test when the test statistic has a χ^2_m distribution.

m	1	2	3	4	8	9	10
Crit Val.	3.84	5.99	7.81	9.48	15.50	16.91	18.30

m	90	91	98	99	100
Crit Val.	113.14	114.26	122.10	123.22	124.34

Matrix Inverse

The inverse of a 2 by 2 matrix

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}^{-1} = \frac{1}{ad - bc} \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

2. Consider the MA(2)-GARCH(1,1) model

$$
\begin{aligned}
y_t &= \phi_0 + \theta_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \epsilon_t \\
\epsilon_t &= \sigma_t e_t \\
\sigma_t^2 &= \omega + \alpha_1 \epsilon_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \\
e_t &\overset{\text{i.i.d.}}{\sim} N(0,1)
\end{aligned}
$$

(a) [10%] What conditions are required for ϕ_0, θ_1 and θ_2 for the model to be covariance stationary?

(b) [10%] What conditions are required for ω,α_1, and β_1 for the model to be covariance stationary?

(c) [10%] Show that $\{\epsilon_t\}$ is a white noise process.

(d) [10%] Are ϵ_t and ϵ_{t-1} independent?

(e) [60%] What are the values of the following quantities:

i. $\mathrm{E}[y_t]$

ii. $\mathrm{E}_t[y_{t+1}]$

iii. $\mathrm{E}_t[y_{t+2}]$

iv. $\lim_{h\to\infty} \mathrm{E}_t[y_{t+h}]$

v. $\mathrm{V}_t[y_{t+1}]$

vi. $\mathrm{V}_t[y_{t+2}]$

3. Consider the VAR(P)

$$\boldsymbol{y}_t = \boldsymbol{\Phi}_1 \boldsymbol{y}_{t-1} + \boldsymbol{\Phi}_2 \boldsymbol{y}_{t-2} + \boldsymbol{\epsilon}_t$$

(a) [15%] Write this in companion form. Under what conditions is the VAR(P) stationary?

(b) Consider the 2-dimentional VAR

$$\boldsymbol{y}_t = \boldsymbol{\Phi}_1 \boldsymbol{y}_{t-1} + \boldsymbol{\epsilon}_t$$

i. [15%] What conditions on Φ_1 are required for the VAR(1) to have cointegration?

ii. [20%] Describe how to test for conintegration using the Engle-Granger method.

(c) [15%] Define conditional Value-at-Risk. Describe two methods for estimating this and compare their strengths and weaknesses.

(d) [15%] Define expected shortfall. Is this a more or less difficult object to estimate than Value-at-Risk? Why?

(e) [20%] Give the formula for the original 1996 RiskMetrics model. How does this differ from the updated 2006 RiskMetrics model? How is this 1996 model estimated?