Economic Factors and the Covariance of Equity Returns

Kevin Sheppard*
Department of Economics
and
Oxford-Man Institute of Quantitative Finance
University of Oxford

This version: February 15, 2008

Abstract

This paper examines equity return covariance using a set of economically meaningful exogenous variables in place of lagged return cross-products. A model is developed which allows for the inclusion of exogenous variables in the conditional covariance without requiring the variables to be non-negative, parameterizing the symmetric square-root of the conditional covariance as a linear process in the explanatory variables. Using a set of 6 market equity and BE/ME sorted portfolios, exogenous variables are able to explain up to 10% of the variation in conditional variances and up to 30% in the variation in conditional correlations. The model is extended to allow for ARCH effects where the effect of exogenous variables, while diminished, remains significant. Further, including exogenous variables lowers the persistence in conditional covariances as measured by the parameter on lagged return cross-products. These findings are consistent with a two time-scale interpretation of conditional covariances, a high frequency component lasting a few months, and a low frequency, business-cycle length component captured by the exogenous explanatory variables.

JEL Codes: C32, C51, G11
Keywords: Correlation, Equity Covariance, Multivariate GARCH, Time-varying Risk, Volatility

1 Introduction

Research into the dynamics of equity volatility has been one of the most active and productive fields of empirical finance over the past two decades. Countless studies have documented time-variation, high persistence and conditional asymmetries in the covariance of many financial asset returns. However, with few notable exceptions, research on financial asset volatility has focused on time-series properties of these series. The vast majority of the literature on time-varying covariance has relied exclusively on lagged cross-products (or functions of cross-products) as the sole driving shock for the dynamics of covariance.

Simultaneously, a significant literature has emerged on both rational and (possibly) irrational predictability of equity returns and factors affecting the cross-sectional dispersion of returns. Using a variety of financial and macroeconomic factors, a number of papers have documented economically meaningful variation

*Mailing address: Department of Economics, Manor Road Building, Manor Road, Oxford, OX1 3PG, UK email: kevin.sheppard@economics.ox.ac.uk. The author would like to thank Robert F. Engle, Graham Elliott and Liangjun Su for helpful discussion. Software and data used in this paper, as well as the latest version of this paper, are available at http://www.kevinsheppard.com. All remaining errors are my own.

in conditional *means* of equity returns. Chen, Roll & Ross (1986) and Jagannathan & Wang (1996) have examined the role macroeconomic factors play in the determining equity premiums. These two papers have shown that certain macroeconomic variables such as the labor income growth rate or unexpected inflation are meaningful determinants of the equity premium or dispersion of the premium. Kiem & Stambaugh (1986), Fama & French (1988) and Kandel & Stambaugh (1990) were among the first to document the predictability of equity premiums using variables shown to be related to the state of the economy, and have since been augmented with important contributions such as Ferson & Harvey (1991), Ferson & Harvey (1999) and Chordia & Shivakumar (2002). These papers have all shown that certain variables such as the dividend yield, short term interest rates or the default spread can explain between 1 and 3% of the short term variation in equity returns.

If these variables are related to non-diversifiable sources of risk, either to the market as a whole or to a subset of assets, it should be the case that they should also be relevant determinants of the conditional covariance of equity returns. Some of the variables with explanatory power over returns have been examined in the context of volatility models, typically in studying the volatility of the market portfolio. Officer (1973), Schwert (1989), Glosten, Jagannathan & Runkle (1993), Whitelaw (1994), and Brandt & Kang (2002) have examined at least one economically meaningful variable in explaining the volatility of returns, although little consensus has been formed on the role these variables play in the covariance of equity returns. Officer (1973) examined the role variability of industrial production plays in determining the volatility of various market indices, finding that there is a significant relationship between the two. Schwert (1989) found little evidence that macroeconomic variables, including bond returns, short term interest rates, producer prices or industrial production growth have incremental information for monthly market volatility over lagged squared returns. Glosten et al. (1993) found evidence that short term interest rates play an important role in future market variance. Whitelaw (1994) found statistical significance for a commercial paper spread and the 1-year treasury rate, while Brandt & Kang (2002) used the short term interest rate, term premium, and default premium as controls in examining the relationship between mean and variance, finding a significant relationship for all three. Other research into the volatility of equities, including Hamilton & Lin (1996) and Perez-Quiros & Timmermann (2001) have found evidence that the state of the economy is an important determinant in the volatility of equity returns. However, the methodology employed in these two studies does permit direct interpretation of the effect of macroeconomic variables on volatility.

The goal of this paper is to examine the role financial and macroeconomic state variables play in the conditional covariance of equity returns. The first issue addressed is to develop an econometric methodology where conditional covariances can be modeled in a meaningful way using exogenous variables. Most attempts to model conditional covariance using exogenous state variables can be classified into one of two types: regression based or exponential GARCH (Nelson 1991). Regression based techniques, used in Officer (1973), Schwert (1989), Whitelaw (1994) and Harvey (2001), are easy to estimate although difficult to interpret as they disregard the structure of the problem and make no attempt to ensure conditional variances are non-negative. Exponential GARCH, used in Glosten et al. (1993), Brandt & Kang (2002) and Harvey (2001), solves the non-negativity problem by modeling the log of variance but is ill-suited to modeling conditional covariances.

The model proposed in this paper addresses both of these issues while preserving the ability to examine the statistical relevance of individual variables. This paper proposes to model the symmetric spectral square-root of the conditional covariance as a linear function of exogenous variables. While the parameters in the square-root are, in general, not directly interpretable, the resulting form for the conditional covariance is a pseudo-linear process in the cross-products of exogenous variables allowing for interpretation in terms of the exogenous variables, their squares and their cross products. The model is extended to allow for ARCH effects where the terms involving exogenous variables can be interpreted as a time-varying intercepts. Finally, the paper shows how the same techniques used to model the conditional covariance with exogenous variables can be used to produce an ARCH model with both time-varying intercepts and time-varying innovation loadings which both depend on possibly different sets of exogenous variables.

Using this modeling strategy, this paper examines the explanatory power of financial and macroeconomic variables for the variance and covariance of a set of six equity returns. The portfolios studied consists of a 2-way sort based on market equity and book-to-market equity, first seen in Fama & French (1993). Examining portfolios rather than the value-weighted market allows for the examination of differing effects of macroeconomic variables on the volatility of firms with different risk exposures. For instance, if the volatility of small firms reacts differently that the volatility of large firms to macroeconomic state variables, the effect on an aggregate market portfolio may be diminished. The variables considered as possible explanatory variables for conditional covariance include the default spread, the term premium, a short term interest rates, changes in the composite leading indicator, industrial production growth, labor income growth, growth in per-capita consumption and a simple expression for unexpected inflation. The paper also considers transformations of these variable which may be more relevant for detecting *changes* in the state of the economy than the present of the state of the economy.

Exogenous variables are capable of capturing some, but not all of the time variation in variances and covariances. However, their effect continues to be significant when ARCH effects are introduced into the model. These results provide evidence of two time-scales in the covariance of returns where state variables are related to slowly evolving changes in covariance. Their inclusion in a model with ARCH effects results in a marked decrease in the persistence of shocks to variance as measured by the coefficients on lagged return squares or cross-products. This finding is related to the notion of structural breaks in covariance, however the treatment used in this paper is more consistent with a smooth evolution of the level of the covariance process.

The paper is organized as follows. Section 2 discusses the econometric methodology used in this paper. Section 3 describes the data and explanatory variables considered, including transformation of the core set of variable which may be more meaningful for examining conditional covariances. Section 4 examines the role of exogenous variables in the variance of six portfolios and section 5 extends these results to the covariance of these returns. Section 6 concludes and discusses topics for future research.

2 Econometric Methodology

This section will describe the econometric methodology necessary to examine conditional covariance using exogenous variables. The foremost challenge in constructing a multivariate volatility model while allowing for exogenous variables is ensuring positive definiteness without sacrificing the ability to interpret the effects of the exogenous variables. Conditional covariance is typically modeled using a (G)ARCH specification such as the diagonal *vech* (Bollerslev, Engle & Wooldridge 1988) were the conditional covariance evolves as a function of lagged squares and cross-products of returns. The evolution of the monthly conditional covariance in a simple multivariate ARCH is given by

$$H_{m+1} = C + A \odot r_m r_m' \tag{1}$$

H_{m+1} is the $K \times K$ conditional covariance of returns in month m, r_m is a $K \times 1$ vector of returns in month m,

C is a $K \times K$ positive semi-definite intercept, A is a $K \times K$ positive semi-definite matrix measuring variance response to shocks and $\odot$ is Hadamard (element-by-element) product. Unfortunately, ARCH models do not provide a natural method to directly include exogenous variables while ensuring positive definiteness of the conditional covariance.

The literature has pursued two strategies for examining the effect of exogenous variables on conditional covariances[1]: modeling the log of variance (Glosten et al. (1993), Harvey (2001) and Brandt & Kang (2002)) or ignoring fundamental properties of conditional covariances and using a least-squares approach (Officer (1973), Schwert (1989) and Whitelaw (1994)). While models of log variance ensure that conditional variance is positive, these models are ill-suited to multivariate applications. The bivariate EGARCH model used in Braun, Nelson & Sunier (1995) is difficult to implement and interpret when $K > 2$, requiring a conditional version of Frisch-Waugh-Lovell to ensure that conditional covariances between shocks are zero. On the other hand, regression analysis is straight forward but cannot ensure that conditional covariances are positive-definite, making interpretation difficult.

A general solution to the problem of ensuring positive definiteness of the conditional covariance without imposing an overly restrictive structure on the conditioning variables is to directly model the square-root of the conditional covariance matrix. All positive (semi-)definite matrices D have a unique (up to sign) symmetric spectral decomposition given by $E\Lambda^{1/2}E'$ where E is a matrix consisting of the eigenvectors of D. Λ is a diagonal matrix of the eigenvalues where the eigenvector in the i$^{\text{th}}$ column of E corresponds to eigenvalue on the i$^{\text{th}}$ diagonal of Λ. Further, $EE' = I_K$ so $E\Lambda^{1/2}E'E\Lambda^{1/2}E' = E\Lambda^{1/2}\Lambda^{1/2}E' = E\Lambda E' \equiv D$. The modeling strategy pursued in this paper will parameterize the square-root of the conditional covariance as a linear process in the exogenous variables. This specification will preserve the ability to test for significance of each explanatory variable as the covariances will have a particular quadratic form.

For example, consider a model of the conditional variance of a single asset return as a function of exogenous variables. In this simple case, the spectral root of the conditional covariance is the square-root of the variance of the return. Writing the square root of the monthly conditional variance as a linear function of N exogenous variables, we have

$$h_{m+1}^{1/2} = B'x_m = \sum_{i=1}^{N} b_i x_{im} \tag{2}$$

and the conditional variance is

$$h_{m+1} = Bx_m x_m' B' = \sum_{i=1}^{N}\sum_{j=1}^{N} b_i b_j x_{im} x_{jm} = \sum_{i=1}^{N} b_i^2 x_{im}^2 + 2\sum_{i=1}^{N}\sum_{j=i+1}^{N} b_i b_j x_{im} x_{jm} \tag{3}$$

where x_m is an $N \times 1$ vector of exogenous variables (possibly including a constant) and B is a $1 \times N$ vector of loadings. The conditional variance is a quadratic form where the loadings on the squared exogenous variables are non-negative and the coefficients on the cross products have either sign. Specifically, when the exogenous variables include a constant and a single explanatory variable, the variance process of h_m will be

$$h_{m+1} = \beta_1^2 + 2\beta_1\beta_2 x_m + \beta_2 x_m^2 \tag{4}$$

[1]While this paper will typically refer to the covariance of returns to be inclusive of the variance and correlation, the literature on the effects of exogenous variables has focused exclusively on variances.

Considering the general problem of modeling the conditional covariance of K assets, the function form appears more complex but retains the same fundamental structure. Assume there are N exogenous variables (including a constant) denoted x_m.[2] Modeling the spectral decomposition as a linear process in x_m:

$$H_{m+1}^{\frac{1}{2}} = B(I_k \otimes x_m) \tag{5}$$

$$= \begin{bmatrix} b_{11} & b_{12} & b_{13} & \dots & b_{1k} \\ b_{12} & b_{22} & b_{23} & \dots & b_{2k} \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{1k} & b_{2k} & b_{3k} & \dots & b_{kk} \end{bmatrix} (I_k \otimes x_m) = \begin{bmatrix} b_{11}x_m & b_{12}x_m & b_{13}x_m & \dots & b_{1k}x_m \\ b_{12}x_m & b_{22}x_m & b_{23}x_m & \dots & b_{2k}x_m \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{1k}x_m & b_{2k}x_m & b_{3k}x_m & \dots & b_{kk}x_m \end{bmatrix} \tag{6}$$

where B is a *block symmetric* matrix of $K(K+1)/2$ blocks of $1 \times N$ parameter vectors, b_{ij}, and $\otimes$ denotes Kronecker product. For simplicity, this paper will use $b_{ij} = b_{ji}$ if $i > j$. In addition, this paper will make use of the following notation for the spectral square-root:

$$B(I_k \otimes x_m) = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_k \end{bmatrix} (I_k \otimes x_m) \tag{7}$$

where B_i is a $1 \times KN$ vector of the form $[b_{i1} \;\; b_{i2} \;\; \dots \;\; b_{ik}]$.

Recent developments in covariance modeling have advocated modeling the Cholesky factor of the conditional covariance as a linear process in the explanatory variables (Andersen, Bollerslev, Diebold & Ebens (2001) and Brandt & Diebold (2003)). However it is important to differentiate between the symmetric spectral root and the Cholesky factor for the purposes of modeling the conditional covariance.[3] The difficulty in using the Cholesky factor arises from the fact that parameter estimates will generally depend on the order of the returns. This is not the case when modeling the symmetric square root. Appendix A discuses this result.

The conditional covariance, H_{m+1}, is

$$H_{m+1} = B(I_k \otimes x_m)(I_k \otimes x_m')B' = B(I_k \otimes x_m x_m')B' \tag{8}$$

which provides a straight forward parameterizations of each element of the conditional covariance as a function of cross products of x_m. For each element of H_{m+1}:

$$h_{ijm+1} = B_i(I_k \otimes x_m x_m')B_j' = \sum_{k=1}^{K} b_{ik} x_m x_m' b_{kj}' = \sum_{k=1}^{K} (b_{ik} \otimes b_{kj})(x_m \otimes x_m) \tag{9}$$

$$= \left(\sum_{k=1}^{K} b_{ik} \otimes b_{kj} \right) (x_m \otimes x_m)$$

This model nests the constant conditional covariance model when x_m only includes a constant. The see this, consider the case where a constant is in the first position and b_{ij1} corresponds to the ij^{th} element of the spectral root of the conditional covariance and the remaining b_{ijk}, $k = 1, 2, \dots$ are zero. The paper will next discuss estimation and then testing.

[2]While there is no requirement to include a constant, ensuring positive definiteness can be difficult when one is not included.

[3]All real positive definite matrices (D) have two square roots (identified up to sign), the Cholesky factor, an upper triangular matrix such that $D = U'U$ and the spectral square-root.

Estimation

Estimation of the model parameters will differ from the standard practice of using cross-products of monthly returns to model monthly volatility in a (Q)MLE framework. Squared monthly returns are noisy signals of the monthly covariance and their use can avoided. The estimation strategy in this paper will employ squares and cross-products of *daily* returns to model monthly covariances in a quasi-maximum likelihood framework. Using sums of returns available at one frequency to estimate the conditional variance at another has been used sparingly in the volatility literature. Many different time period have been considered by Officer (1973) (annual constructed from monthly), Schwert (1989) (monthly constructed from daily), and Andersen et al. (2001) (daily constructed from intradaily). Merton (1980) has provided a theoretical justification for this construct. However, in order to treat volatility as an a observable variable, it is necessary to be able to sample infinitely often, which is not the case when using daily observations to construct monthly volatility.

Daily returns (r_t, a $K \times 1$ vector) are assumed to be conditionally multivariate normal with mean zero and conditional covariance H_{t+1} given the information up to time t.

$$r_{t+1}|\mathcal{F}_t \sim N(0, H_{t+1}) \tag{10}$$

where $\mathcal{F}_t$ is the time t information set. The log likelihood of the data and parameters is given by

$$LL(r_t, B) = -\frac{1}{2}\sum_{i=1}^{T} k\ln(2\pi) + \ln(|H_t|) + \text{tr}(H_t^{-1} r_t r_t') \tag{11}$$

where tr is the trace operator (sum of diagonal elements). This form is equivalent to the standard log-likelihood as $\text{tr}(\Sigma_t^{-1} r_t r_t') = \text{tr}(r_t'\Sigma_t^{-1} r_t) = r_t'\Sigma_t^{-1} r_t$. An equivalent log likelihood can be written by grouping the days into M months, each with D_m days replacing the time subscript t by md to indicate month m, day of month d.

$$\begin{aligned} LL(r_t, B) &= -\tfrac{1}{2}\left(\textstyle\sum_{m=1}^{M}\sum_{d=1}^{D_m} k\ln(2\pi) + \ln(|H_{md}|) + \text{tr}(H_{md} r_{md} r_{md}')\right) \\ &= -\tfrac{1}{2}\textstyle\sum_{m=1}^{M} D_m\left(k\ln(2\pi)\right) + \sum_{d=1}^{N_m}\ln(|H_{md}|) + \text{tr}(H_{md} r_{md} r_{md}') \end{aligned} \tag{12}$$

To transform this model of daily returns into one for monthly variance, the model will replace daily covariance in a particular month with the average daily variance within that month. Let $\bar{H}_m$ denote the average variance in month m. Using this substitution, the model can be rewritten as

$$\begin{aligned} LL(r_t, B) &= -\tfrac{1}{2}\textstyle\sum_{m=1}^{M} D_m k\ln(2\pi) + \sum_{d=1}^{D_m}\left(\ln(|\bar{H}_m|) + \text{tr}(\bar{H}_m^{-1} r_{md} r_{md}')\right) \\ &= -\tfrac{1}{2}\textstyle\sum_{m=1}^{M} D_m k\ln(2\pi) + D_m\ln(|\bar{H}_m|) + \sum_{d=1}^{D_m}\text{tr}(\bar{H}_m^{-1} r_{md} r_{md}') \\ &= -\tfrac{1}{2}\textstyle\sum_{m=1}^{M} D_m k\ln(2\pi) + D_m\ln(|\bar{H}_m|) + \text{tr}(\sum_{d=1}^{D_m}\bar{H}_m^{-1} r_{md} r_{md}') \\ &= -\tfrac{1}{2}\textstyle\sum_{m=1}^{M} D_m k\ln(2\pi) + D_m\ln(|\bar{H}_m|) + \text{tr}(\bar{H}_m^{-1}\sum_{d=1}^{D_m} r_{md} r_{md}') \\ &= -\tfrac{1}{2}\textstyle\sum_{m=1}^{M} D_m k\ln(2\pi) + D_m\ln(|\bar{H}_m|) + \text{tr}(\bar{H}_m^{-1} RV_m) \\ &= -\tfrac{1}{2}\textstyle\sum_{m=1}^{M} D_m\left(k\ln(2\pi) + \ln(|\bar{H}_m|) + \text{tr}(\bar{H}_m^{-1}\frac{RV_m}{D_m})\right) \\ &= \textstyle\sum_{m=1}^{M} D_m\left[\text{LL}_m(\frac{RV_m}{D_m}, \bar{H}_m)\right] \end{aligned} \tag{13}$$

where RV_m is the monthly realized covariance[4] constructed by summing the cross products of the daily

[4]The expression *realized covariance* has taken on a special meaning over the past few years. Andersen & Bollerslev (1997) have defined the realized covariance as integrated volatility constructed using high frequency returns. However, the moniker is also appropriate here as the monthly covariances used in this paper are a coarse approximation to the quadratic variation of returns. However, with such a coarse grid for summing and squaring the returns, it is not appropriate to treat the realized covariances as observable variables, motivating the use of (Q)MLE.

return vectors. In essence, the log-likelihood for average monthly covariance is a weighted log-likelihood where the weights are the number of days in the months. One particular crucial assumption is required for daily cross-products of returns to be a meaningful measure of the covariance: that daily returns are zero mean martingales conditional on past returns. Scholes & Williams (1977) have shown that this property will not necessarily hold for individual issues when considering closing prices if stocks trade at different frequencies. For instance, the return of an infrequently traded issue may be correlated with the past return of a frequently traded issue if the latest price of the frequently traded stock reflects up to the minute news while the price of the infrequently traded issue may not reflect the news until the next trading day. Schwert (1989) explicitly considered this problem for the value weighted market returns by using an estimator which accounted for this potential autocorrelation structure. Specifically, the variance of returns in month m was constructed using:

$$RV_m = \sum_{d=1}^{D_m} r_{md} r'_{md} + r_{md} r'_{md-1} + r_{md-1} r'_{md} \tag{14}$$

This estimate of the monthly covariance is not guaranteed to be positive semi-definite. However, a simple Newey-West estimator can be used in place of Schwert's estimator, changing the monthly realized covariance estimator to

$$RV_m = \sum_{d=1}^{D_m} r_{md} r'_{md} + \frac{1}{2}(r_{md} r'_{md-1} + r_{md-1} r_{md}) \tag{15}$$

Neither of these estimators for the monthly realized covariance includes a model for the conditional mean. There are two justifications for omitting the model for the conditional mean, one theoretical and the other practical. Theoretically, if a return $r_m \sim D(\mu_m, H_m)$ in a month with D_m days, assuming the mean is constant throughout the month, summing and squaring the daily return will produce an estimator of the variance with a bias which is $o(D_m)$. For instance,

$$E[\sum_{d=1}^{D_m} r^2_{dm}] = \sum_{d=1}^{D_m} \left(\frac{\mu_m}{D_m}^2\right) + \frac{H_m}{D_m} = H_m + \frac{\mu_m}{D_m} = H_m + o(D_m) \tag{16}$$

Thus, if the number of days in the month (or observations of the return process) becomes large, the mean will be negligible. Unfortunately, an average of 21 days in a month may not be sufficiently high to satisfy this theoretical justification. However, from a practical perspective, the R^2 from most predictive regression for return is less then 3%, so the effect of a neglected mean in a raw monthly return (ignoring the decrease from using a realized method) should explain be less than .1% of the movements in the squares.

However, it is a simple extension to allow for a conditional mean in the realized estimator. Suppose $r_m \sim D(\mu_m, H_m)$. It is easy to define a demeaned daily return as $\tilde{r}_{md} = r_{md} - \frac{\mu_m}{D_m}$ and to construct the realized volatility using either the simple or the Newey-West estimator from the $\tilde{r}_{md}$. In the empirical portion of this paper, the bulk of the estimation will use the raw returns although the effect of conditional mean dynamics will be explored.

Issues relating to the choice of measurement of realized covariance will be examined in the context of the data used in this paper in section 3.

Testing

The ultimate goal of this model is to be able to examine the statistical significance of exogenous variables as a leading variable in conditional covariances. Under certain regularity conditions, White has established consistency and asymptotic normality of quasi-maximum likelihood estimators (White 1996). Under these conditions

$$\sqrt{M}(vech(\hat{B}) - vech(B_0)) \overset{A}{\sim} N(0, \Gamma^{-1}\Sigma\Gamma^{-1}) \tag{17}$$

where $vech$ is a block half vector operator such that

$$vech(B) = [b_{11}\ b_{12}\ b_{13}\ \ldots\ b_{1k}\ b_{22}\ b_{23}\ \ldots b_{2k}\ \ldots\ b_{kk}]' \tag{18}$$

and each block, $b_{ij} = [b_{ij1}\ b_{ij2}\ \ldots\ b_{ijn}]$, $i \leq j$, $i = 1, \ldots, k$, is $1 \times n$. In the asymptotic covariance, Γ is the expectation of the second derivative of the log-likelihood and Σ is the long-run covariance of the scores of the log-likelihood. Γ is estimated using the sample analog to the expectation of the second derivative of the log-likelihood and Σ is estimated using a Newey-West estimator to account for potential model misspecification. The number of lags was set to 12. Analytical expression for these derivatives are contained in appendix B.

Testing exogenous effects in a univariate specifications is straight forward. From the conditional variance in the univariate specification,

$$h_{m+1} = Bx_m x_m' B' = (b_1 x_{1m} + b_2 x_{2m} + \ldots + b_n x_{nm})^2 \tag{19}$$

A linear hypothesis tests can be conducted on the estimated coefficients to determine whether a variable (or group of variables) is statistically different from zero. For instance, to determine whether the coefficient on x_{nm} is statistically different from zero, the test statistic can be constructed using

$$T = \hat{b}_n \hat{V}(b_n)^{-1} \hat{b}_n \tag{20}$$

where $\hat{V}(b_n)$ is the estimated variance of b_n. This test statistic, T, will be asymptotically distributed χ^2_1.

In a multivariate model, some of the tests require non-linear restrictions although variance and joint significant continue to require only linear restrictions on the parameters. The only tests which remain obviously linear restrictions of the parameters are those which examine whether all parameters on an particular exogenous variable (or set of variables) are zero. This test, which is formally stated for testing that x_{nm} has no effect on covariance as $H_0 : b_{11n} = b_{12n} = \ldots = b_{KKn} = 0$. Under the null, x_{nm} will not appear in any of the conditional covariances. A test statistic based on a Wald statistic will have a $\chi^2_{K(K+1)/2}$ distribution.

Other linear tests are not necessarily meaningful as each term in the conditional covariance is the sum of K quadratic forms, each of which involve all of the cross-products of the exogenous variables x_m. For instance, the conditional covariance between the i^{th} and j^{th} series in a k dimension problem is given by

$$H_{ijm} = B_i(I_k \otimes x_m x_m')B_j' = \sum_{k=1}^{K} b_{ik} x_m x_m' b_{k1} \tag{21}$$

A test whether a variable contains relevant information will require testing that the coefficients on all terms involving that variable are zero. With N exogenous variables, the test will involve N nonlinear restrictions on the parameters. To test whether the n^{th} variable is irrelevant (indicated by all terms involving x_{nt} having coefficients of zero) for the covariance between asset i and asset j, the necessary restrictions are:

$$r(B) = \begin{bmatrix} \sum_{k=1}^{K}(b_{ik1}b_{kjn} + b_{ikn}b_{kj1}) \\ \sum_{k=1}^{K}(b_{ik2}b_{kjn} + b_{ikn}b_{kj2}) \\ \vdots \\ \sum_{k=1}^{K}(b_{ikn-1}b_{kjn} + b_{ikn}b_{kjn-1}) \\ \sum_{k=1}^{K} b_{ikn}b_{kjn} \\ \sum_{k=1}^{K}(b_{ikn+1}b_{kjn} + b_{ikn+1}b_{kjn}) \\ \vdots \\ \sum_{k=1}^{K}(b_{ikN}b_{kjn} + b_{ikn}b_{kjN}) \end{bmatrix} = 0. \tag{22}$$

A test statistic can be formed using a Wald test where the asymptotic variance under the null can be computed using the delta method.

$$T = r(\hat{B}) \left(J(\hat{B})\hat{V}(B)J(\hat{B})' \right)^{-1} r(\hat{B})' \tag{23}$$

where $J(\hat{B}) = \frac{\partial r(\hat{B})}{\partial vech(B)}$ is the jacobian of the restriction vector with respect to the model parameters. Under the null, this test statistic will have be asymptotically distributed $\chi^2_{\text{rank}(J(\hat{B}))}$. In this particular test, the distribution would be a χ^2_N.

However, in the case where $i = j$, the rank, under the null, of $J(\hat{B})$ will necessarily be zero and the limiting distribution of a test using the delta method is nonstandard. Simplifying the above set of restriction of the case $i = j$,

$$r(B) = \begin{bmatrix} \sum_{k=1}^{K} 2(b_{ik1}b_{ikn}) \\ \sum_{k=1}^{K} 2(b_{ik2}b_{ikn}) \\ \vdots \\ \sum_{k=1}^{K} 2(b_{ikn-1}b_{ikn}) \\ \sum_{k=1}^{K} b_{ikn}^2 \\ \sum_{k=1}^{K} 2(b_{ikn+1}b_{ikn}) \\ \vdots \\ \sum_{k=1}^{K} 2(b_{ikN}b_{ikn}) \end{bmatrix}. \tag{24}$$

The difficulty arises from the restriction $\sum_{k=1}^{K} b_{ikn}^2 = 0$. All restrictions will valid if and only if $b_{ikn} = 0$, $k = 1, 2, \ldots, K$. In addition, if $b_{ikn} = 0$, $i = 1, 2, \ldots, K$, the other $n - 1$ restriction are also satisfied as each involves a product of b_{ikn} and another variable. Thus, a linear hypothesis test can be used to examine the significance of an explanatory variable for a conditional variance, $H_0 : b_{i1n} = b_{i2n} = \ldots = b_{ikn}$.

Augmented ARCH

The model, as described, only allows the inclusion of exogenous variables. However, this model can be augmented into an ARCH(Q) framework by including lagged realized covariances (or return cross-products) in addition to the exogenous variables. This modified model, when a constant is included, will nest both the ARCH specification and the strictly exogenous specification:

$$H_{m+1} = B(I_k \otimes x_m x_m')B' + \sum_{q=1}^{Q} A_q \odot RV_{m-q+1}' \tag{25}$$

where A_q are positive semi-definite matrices.[5] When x_m contains a constant, it is necessary to remove C (from equation 1) to ensure identification. In this model, an alternative interpretation of the term with exogenous variables is that of an ARCH specification with time-varying intercepts.

While not a focus of this paper, the techniques used in constructing an exogenous model can be further integrated an ARCH specification through the innovation parameters by scaling the cross products of returns by exogenous variables. Ledoit, Santa-Clara & Wolf (2002) have shown that the innovation parameter matrix in an ARCH model must be positive semi-definite to ensure that conditional covariances are positive definite. Combining the exogenous variables approach to modeling covariance, we can construct an ARCH-like model which allows for both time variation in the intercept and time variation in the innovation parameters. In this model, let x_{1m} by an $N_1 \times 1$ vector of exogenous variables to be included in the intercept and let x_{2m} by an $N_2 \times 1$ vector of exogenous variables to be included in the innovation loadings. In the ARCH(1) case, the conditional covariance can be modeled as

$$H_{m+1} = B(I_k \otimes x_{1m}x'_{1m})B' + A(I_k \otimes x_{2m}x'_{2m})A' \odot RV_m \tag{26}$$

where A is a $K \times KN_2$ block symmetric analog of B (equation 7). In the estimation of model with ARCH effects, the specification of equation 26 will be employed with $[x_{2m}] = [1]$ to trivially ensure positive definiteness of the realized covariance loadings.

3 Data and Descriptive Statistics

This section will describe both the returns and the exogenous variables and provide summary statistics. The portfolios examined are the 6 Fama-French portfolios formed using a 2 by 3 sort of returns based on market equity and book-to-market equity ratios as in Fama & French (1993). At the end of June each year, the median market equity is computed using the median market equity of NYSE firms. The 30% and 70% book-to-market equity quantiles are constructed at the end of June using book equity from the previous fiscal year divided by market equity at the end of the previous December. The six portfolios are formed by intersecting these two sets of breakpoints.[6] The portfolios are (Market Equity - BE/ME ratio):

- Small-Low (SL)
- Small-Medium (SM)
- Small-High (SH)
- Big-Low (BL)
- Big-Medium (BM)
- Big-High (BH)

Data was available daily from July 2, 1963 until December 31, 2003, a total of 9944 daily observations in 474 months. In addition to examining the dynamics of covariance using monthly data, quarterly realized variances will be considered resulting in 158 quarters over the same period.

The dependant variable for examining covariance is the monthly (quarterly) realized covariance constructed each month (quarter) by summing the cross-products of daily returns during the month (quarter).

[5] Typically, lagged cross-products of returns are used. However, this paper will make use of lagged realized covariance in place of lagged return cross-products.

[6] The daily returns on the 2 by 3 sort were made available by Ken French. The French data library is available at: `http://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html`.

$$RV_m = \sum_{d=1}^{D_m} r_{md} r'_{md} \tag{27}$$

for the monthly realized covariances where D_m is the number of days in month m and

$$RV_q = \sum_{d=1}^{D_q} r_{qd} r'_{qd} \tag{28}$$

for the quarterly realized covariance where D_q is the number of days in quarter q. When working with equities which trade at potentially different frequencies such as small and large market capitalization stocks, cross-autocorrelations can arise between frequently traded issues and those which trade less frequently as noted in Scholes & Williams (1977). This phenomena appears to be less of a problem when using well diversified portfolios than with individual issues using daily data. The first cross-autocorrelation ($Corr(r_{id}, r_{jd-1})$) for the all six of the portfolios with respect to any of the other five portfolios was positive and statistically significant. However, with 9944 daily observations, statistical significance isn't surprising. Inclusion of lagged cross-products does not affect the dynamics of the covariance, instead representing a small level shift while increasing the variance of the measured covariances considerably. Further, there is no appreciable difference in the results of this paper whether the realized covariance estimator without lags is used or the Newey-West estimator described in section 2 is employed. In the spirit of simplicity, this paper will primarily make use of the monthly realized estimator which uses daily cross-products without lags, although the effect of a Newey-West style estimator will be considered.

The choice of this set of portfolios was motivated by two concerns. First, it is important to choose portfolios which are sufficiently diversified that the idiosyncratic portion of the returns will not drown common systemic portions of volatility. These portfolios are all well diversified, particularly in recent years. The least diversified, Big-High, has between 100 and 200 equities during the sample. The number of equities in the small market equity portfolios ranged from the low hundreds to greater than 1000 over the course of the sample and the remaining large market equity portfolios typically contained more then 200 issues. The second motivating factor was to chose a sort whose portfolios would potentially reflect different exposures. Numerous studies have documented asymmetric exposure of small and large firms to changes macroeconomic level of activity. While previous studies have failed to find a strong link between explanatory variables linked to the macroeconomic activity and stock market volatility, it is possible that the use of market portfolios make it more difficult to find significant relationships. For instance, if small firms and large firms have different exposures at one point in time, these may cancel out providing misleading evidence that macroeconomic factors do not matter. These six allow for the examination of effects of exogenous variables along both dimensions of the sort by comparing estimates of the small market equity portfolios with the large market equity portfolios and examining the relevance of the explanatory variables as BE/ME changes.

[INSERT TABLE 1 HERE]

Table 1 contains summary statistics of the six portfolios. The top panel contains annualized mean, standard deviation, as well as skewness and kurtosis calculated from both daily and monthly returns. Returns are generally higher for small firms than for large firms, and returns are increasing in BE/ME ratios. Variances are typically lower for small firms and are lower for high BE/ME firms. Both kurtosis and skewness tend increase as the BE/ME ratios increase and kurtosis is higher for large firms than for small firms and were closer to normality for monthly returns than for daily returns. The bottom panel contains annualized standard deviations as well as correlation of daily returns. Average correlations are generally high and are highest for elements closer to the diagonal (i.e. for portfolios which are the most similar) with the obvious exception of the correlation between the SH and BL portfolios. Returns on portfolios formed on small firms are more correlated with returns on other small firm based portfolios than large firms are with other large firm portfolios, although the correlations within both groups are typically in excess of 90%.

[INSERT FIGURE 1 HERE]

Average statistics cannot adequately provide an overview of the dynamics of variance and correlation. Figure 1 contains plots of the realized variances of the six returns transformed to annualized standard deviations as well as an MA(12) of the annualized standard deviation. There is a significant amount of comovement in the variances evidenced in the graphs and the realized variance series are highly correlated. The minimum correlation of the realized variances was 82% and more than half of the realized variances had correlations in excess of 90%. The variances tend to increase during recessions (NBER recession dates are indicated by shaded regions) and exhibit considerable variation over the sample. All of the variance series clearly reflect the increased variance of the late 1990s (see Campbell, Lettau, Malkeil & Xu (2001) for an exposition).

Figure 2 contains plots of the 15 realized correlations and an MA(12) of the realized correlations. The correlations between portfolios in the same market equity group tend to be less dynamic than correlations between sized based groups and are decreasing in the average level of correlation. The correlation between portfolios which contain the small market equity stocks were particularly non-dynamic. Correlations are generally high and slowly time varying. The lowest correlations occurred during the mid to late 1990s, particularly for the BH portfolio and correlations were never negative. Unlike the variances, there is no obvious patterns evident during (or immediately before or after) recessions.

[INSERT FIGURE 2 HERE]

Figures 3 and 4 contain autocorrelograms of the realized variances and realized correlations, respectively. Figure 3 also contains the autocorrelogram of the log of realized variance. The realized variances during October 1987 (and to a lesser extent November 1987) were so large that most autocorrelations are insignificant outside of the first few. The use of log-variance addresses this problem as October 1987 is much less of an outlier in this transformed series.[7] The autocorrelations of log realized variance show a strong cyclical pattern of almost 7 years. The high BE/ME portfolios, and to a lesser extent the small market equity portfolios, show evidence of a shorter cycle near 4 years. The autocorrelograms of the realized correlation show less persistence than the those of the variances, but there is still evidence of 12 to 24 months of significant autocorrelations in each of the correlation series. The weakest evidence for time-variation in correlation is between the small equity portfolios while the greatest is for the BH portfolio.

[INSERT FIGURES 3 AND 4 HERE]

The choice of explanatory variables is primarily motivated by previous work in examining predictable components in conditional means or unconditional changes in equity returns. This choice is motivated by the notion that these variables are capturing a form of non-diversifiable risk which should be evidenced in conditional variances.

[7] When a dummy is used for October 1987, the autocorrelogram of monthly realized variances is similar to that of log realized variances.

Composite Leading Indicators (%ΔCLI): The composite leading indicator is weighted average of ten macroeconomic series thought to lead economic activity. The CLI contains a time trend and is possibly integrated. The CLI is transformed to a growth rate to address these problems. The data series, DLEAD, is originally form The Conference Board and was taken from the DRI-BASIC database.
Default Premium(DEF): The default premium is measured as the spread between BAA and AAA rated bonds. DEF is constructed using Moody's seasoned corporate bond indices from the FRED II database.
Industrial Production(%ΔIP): Percent change in industrial production index measured as the log difference of the industrial production index. The data are from the Federal Reserve Board of Governors G.17 release via the FRED II database.
Per-Capita Labor Income Growth(%ΔLBR): Percent change in per-capita labor income measured as the log difference of per-capita labor income. The data are from the FRED II database.
Per Capita Consumption (%ΔPCC): Percent change in per-capita consumption is measured as the log difference of nondurable per-capita consumption. Per-capita consumption was constructed by dividing seasonally adjusted personal non-durable consumption by the mid-month US population. Both aggregate consumption and mid-month population (respectively PCEND and POPTHM) are from the Department of Commerce: Bureau of Economic Analysis via the FRED II database.
Short Term Interest Rate (SHORT) The short term interest rate is the yield on a 3 month treasury. The actual rate was taken from the DRI-BASIC (formerly Citibase) database series FYGM3.
Term Premium (TERM): The premium for holding long term treasury bonds over short term bonds, calculated as the difference in the interest rates on a 10 and a 1 year treasury note. Both rates were taken from the Federal Reserve H.15 publication via the FRED II database.
Unexpected Inflation (UINF): A simple measure of unexpected inflation is calculated as the monthly percent change in seasonally adjusted CPI (without energy) minus the average change over the past 12 months ($\text{UINF}_m = \ln(CPI_m) - \ln(CPI_{m-1}) - \frac{1}{12}\sum_{i=1}^{12} \ln(CPI_{m-i}) - \ln(CPI_{m-i-1})$). The CPI data series, CPILEGSL, is from the U.S. Department of Commerce: Bureau of Economic Analysis via the FRED II database.

The data were all from July 1963 until December 2002, a total of 474 months and 158 quarters. Quarterly data were constructed as the last monthly observation from the month immediately before the start of a new quarter and quarterly growth rates were quarter over quarter. Two series which are often included in the list of explanatory variables are missing from this list: the market return and the dividend yield. The goal of this paper is to examine the covariance of returns without explicitly employing squared returns, so the market return must be excluded. In addition, it has been noted that the use of the dividend yield, as well as any other series which contains the market value of equity, can lead to dubious conclusions. For instance, in the short term, nearly all changes in the yield are driven the market value of equity. In practice, interpreting the dividend yield is particularly difficult as it is more of a signal of the recent level of the market than an indicator of the prospects for future growth and is essentially a bear market dummy.

Three of the explanatory variables have become standard explanatory variables for examining the mean of returns. The default premium, the term premium, and the short term interest rate have been widely used in predictability studies including Ferson & Harvey (1991) and Chordia & Shivakumar (2002) among others.[8] Perez-Quiros & Timmermann (2001) used changes in the CLI as a state variable for modeling the transition probabilities between high and low volatility states using size sorted portfolios. Officer (1973) and Hamilton & Lin (1996) examined the relationship between industrial production growth and market volatility, finding significant evidence that these two variables move together. Consumption growth, while it hasn't

[8]Most of these studies include the dividend yield and/or lagged returns. However, for the stated reason, these are not included in this study.

been explicitly used to predict conditional mean of equity returns, has a strong theoretical relationship to equity returns. Labor income was found by Jagannathan & Wang (1996) to be an important explanatory variable in the cross-section of equity returns. Unexpected inflation was employed by Chen et al. (1986) as an explanatory variable for changes in the market return.

Figure 6 contains a plot of these 8 exogenous series. It is obvious that these variables have different degrees of persistence, although all exhibit some persistence. The least persistent of the series are the growth series (%ΔIP,%ΔLBR %ΔPCC) and the measure of unexpected inflation used in this paper (UINF). The growth of the composite leading indicator (%ΔCLI) is more persistent, although it is less persistent than any of the three financial series. DEF, SHORT and TERM are all highly persistent. However, it should be noted that low persistence in the level of a variable does not necessarily mean that it cannot have a persistent effect on a conditional covariance. The structure of the model results in conditional covariances which are linear in the squares and cross-products of the explanatory variables. Hence a variable persistent in its square (but not in mean) can still be useful in explaining persistence in return covariance persistence.

Transformations of exogenous variables

The preliminary choice of has been motivated by previous examinations of the mean and cross-sectional dispersion of equity returns. However, in the structure of this model, it is not obvious that the particular transformations used for return modeling are appropriate for examining covariances. For instance, while a high default premium is typically associated with increased risk for small firms risk, it may not be the case that volatility will be by high when small firm risk is high. It may be the case that volatility is high when uncertainty about the state of the economy is high, for example when the default premium has changed in the recent past. Veronesi (1999) has constructed a model where a number of stylized fact about the volatility of equity returns can be explained. Modeling the state of economy as a latent process, Veronesi finds that volatility is highest when the amount of uncertainty about the state of the economy is highest. In an effort to allow variables which can capture a changing state in the economy, this paper will consider a number of alternative transformation of the 8 explanatory variables.

This paper will consider 9 transformations of the 8 original variables. The majority of these will be a transforation into one-year changes. These are better capable of capturing changes in the state of the economy in the framework in this paper than either levels (capture the state, not changes in the state) or percent changes are (capture only the most recent change). The first seven new variables correspond to the seven explanatory variables transformed into annual changes.[9]

$\mathbf{CLI}_{\mathbf{1YR}}$: Difference of the log of the composite leading indicator between months m and $m-12$.
$\mathbf{DEF}_{\mathbf{1YR}}$: Difference between the default spread between months m and $m-12$.
$\mathbf{INF}_{\mathbf{1YR}}$: Difference between the log consumer price index between months m and $m-12$
$\mathbf{IP}_{\mathbf{1YR}}$: Difference of the log of the industrial production index between months m and $m-12$.
$\mathbf{LBR}_{\mathbf{1YR}}$: Difference between log of per-capita labor income between months m and $m-12$.
$\mathbf{PCC}_{\mathbf{1YR}}$: Difference of the log of per capita consumption between months m and $m-12$.

[9] One-year and monthly changes of the short term interest rate are not included as these are highly correlated with one year and monthly changes in the term premium. In essence, most of the volatility of the term premium arises from changes in the short rate and the 1 year and 3 month rates are highly correlated.

TERM$_{1YR}$: Different between the term premium between months m and $m - 12$.

Also considered will be the short term changes of two of these variables. Short term variations of other variables were either already included, had a near equivalent included or do not make sense.

ΔTERM: Change in the term premium between t and $t - 1$
ΔDEF: Change in the default premium between time t and time $t - 1$.

These 17 variables compose the set of exogenous variables which will be used throughout the remainder of this study. Table 2 contains a summary of the exogenous variables.

4 Variance Results

This section will present results for the variance series each of the six return series modeled separately. While the specification in this paper is explicitly multivariate, interpretation of univariate results is less complicated and is useful for developing intuition as to which variables are important determinants of conditional covariances.

The first model estimated examines the relationship between the eight core explanatory variables and the six portfolio variances. The log-likelihood is only identified up to a constant and this paper will make use of a normalization where the first element of B is positive. Table 3 contains results for the strictly exogenous model which included a constant and the eight core explanatory variables. The conditional covariance for each of these series is given by:

$$h_{m+1} = Bx_m x_m' B' = (b_1 x_{1t} + b_2 x_{2t} + \ldots + b_n x_{nt})^2 \tag{29}$$

The explanatory variables enter the conditional covariance in a non-linear fashion. In order to allow for the interpretation of the parameters, it is necessary to consider a linearization of the effect of the explanatory variables. The tables will contain the average partial effect on the annualized standard deviation of a change in the explanatory variable. Mathematically, given a series of conditional covariances $h_1, h_2, \ldots, h_M$, this expression is given by

$$\overline{PE}(x_i) = \frac{1}{M}\sum_{m=1}^{M} \frac{\partial(\sqrt{12 h_m(x_m)})}{\partial x_i} = \frac{\sqrt{3}}{M}\sum_{i=1}^{M} \frac{\partial h_m}{\partial x_i} h_m^{-\frac{1}{2}} \tag{30}$$

In addition to the average partial effects, p-values for the null the effect of the variable is zero, H_0 : $b_m = 0$ are reported in parenthesis below each partial effect (p-values based on estimated coefficients). Most of the model parameters are statistically significant although the growth in per-capita consumption is never significant using a 10% test. The default premium (DEF) is the only variable which is statistically significant for all of the series, typically with a p-value below .001. The average partial effects (APE) in all of the models on the default premium are positive and are larger for large firms than for small firms. The default premium has a strong relationship to business cycles and the differences in the loadings indicates that the volatility of large firm is more sensitive to this business cycle indicator. Changes in the CLI (%ΔCLI) are significant leaders of volatility for 4 of the series and had p-values less than 30% for the series where insignificant. All of the estimated effects are negative, indicating that decreases in the leading indicator correspond to higher conditional variances. There is also a strong asymmetry in the in response to growth in the CLI. APEs are considerably larger (in absolute value) for the large market equity portfolios than for the small equity portfolios. This consistent with the idea that the volatility of large corporations are more responsive to macroeconomic news than are small firms.

[INSERT TABLE 3 HERE]

Unexpected inflation (UINF) is statistically significant for 3 of the 6 variance series and is only marginally insignificant (p-values between 10% and 13%) in the other three series. The coefficients are positive implying that unexpected inflation increases volatility of these series. However, unexpected inflation, as defined in this paper, has virtually disappeared beginning in the 1990s. The significance of this result must be interpreted with caution as the impact of unexpected inflation must have occurred during the first two-thirds of the sample.

Two of the variables have strong, significant effects for small firms but are generally insignificant for large firms. The short rate (SHORT) is significant for all 3 of the small ME portfolios and also for the BH portfolio. All of the APEs are negative indicating that high short rates correspond to decreased volatility of these portfolios. Short rates are usually highest when the economy is strong or inflation is high, and lowest when the economy is weak. Short rates are decreasing during all of the recession in the sample. The term premium is also significant for the small market equity firms and insignificant for the large market equity firms. The negative coefficient indicates that volatility will be decreasing when the term premium is largest and increasing when the yield curve is inverted. The term premium has a two interpretations. A negative term premium corresponds to a short term rate (1 year) which is higher than a longer term rate (10 year). One potential cause of a negative term premium is high inflation. In this interpretation inflation would have a positive effect on volatility. An alternative interpretation is that the market has anticipated changing economic conditions while the monetary authority hasn't reacted quickly enough. In this interpretation, the negative short rate will correspond to impending economic difficulties. In addition, the average partial effect of the term premium is more negative the APE of the short term rate, indicating that increases in the short rate ultimately lead to higher volatility. Industrial production growth is also significant for small firms, while labor income growth shows marginal significance for large firms.

The table also contains a test that all parameters on the explanatory variables are zero. Formally, this null can be stated $H_0 : b_2 = b_3 = \ldots = b_8 = 0$. All models reject this null with a p-value below 0.0005, providing strong evidence of the role these explanatory variables play in the volatility of these returns. One of the explanatory variables was insignificant in all six models. Growth in per-capita consumption (%ΔPCC) had no coefficients which were statistically significant in any of the models using a 10% test. Aggregate macroeconomic variables, specifically consumption, have had a difficult time explaining equity returns (Campbell & Cochrane 2000) so these results are not particularly surprising. In addition, per-capita consumption is remarkably smooth, providing little to differentiation it from a constant.

[INSERT FIGURE 6 HERE]

Figure 6 contains a plot of the realized volatility and the fit volatility for these 6 series (all series have been transformed to annualized standard deviations). From the graphs, it is obvious that the fit volatilities are not nearly as dynamic as the realized volatilities. There are long periods where the realized volatility is above or below the fit volatility, particularly in between 1963 and 1966 and in the low volatility period after October 1987. However, the fit volatility series do appear capable to picking up general trends in volatility such higher volatility near and during recessions (as indicated by shaded regions) and the increase in volatility in the past decade.

Table 4 contains the results of fitting the strictly exogenous model to all six variance series using the expanded set of 17 explanatory variables. Comparing the results of Table 3 with those of Table 4, most of the variables which were significant in the smaller models continue to be significant at the 10% level. In addition the sign on all of the APEs included in both the core model and the expanded model agree. A number of the transformed variables are significant. The one year change in the composite leading indicator (CLI_{1YR}) is significant in 5 of the 6 models and shows a strong asymmetry as BE/ME increases. Percent change in the composite leading indicator is now only significant for large firms; the only change in significance between the small and large models. The signs on both of APEs are all negative, indicating that decreases in the composite leading indicator over either the monthly or the annual horizons increase volatility. The default rate is significant for all series while the one year change in the default spread (DEF_{1YR}) is significant for the BL and the BH portfolios and the monthly change in the default spread (ΔDEF) is significant for the BM and BH portfolios. Unexpected inflation is significant for all of the variance series and the year over year inflation is significant for 5 of the 6 portfolio variances. The effect of the one year change in inflation on the variances is strongly asymmetric across BE/ME, evidencing a considerable decrease as BE/ME increases. The one year change in labor income is significant in for the three large firm series. One of the industrial production index derived series and two of the per-capita consumption derived series were significant.

[INSERT TABLE 4 HERE]

The short rate continues to be significant for the small market equity portfolios. Both the term premium and the one year change in the term premium are significant for the variance of all portfolios. In addition, the one month change in the term premium is significant for 5 of the 6 portfolios. However, the sign on the one year change is negative while the sign on the APE of the premium and the one month change are negative. However, the APE of the one month change is larger than the effect ont he one year indicating that move toward a normal upward term premium should lower variance. The two last lines in the table contains a test that the coefficients on all of the explanatory variables are zero. All models strongly reject this null.

Both the original set of explanatory variables and the expanded set present strong evidence that certain financial and macroeconomic variables, particularly the composite leading indicator, inflation, the default rate, the short rate and the term premium have explanatory power for the variance of the six portfolios. However, it is difficult to interpret these results in the context of the literature which has made use of various ARCH model specifications. The obvious question is whether the variables which were significant in the strictly exogenous models were significant because they are highly persistent and only picking up on information contained in lagged squared returns or do they represent sources of volatility not contained in lagged squares. To address this issues, the paper will examine these findings in a model which includes both the explanatory variables and ARCH effects. The conditional covariance in a model with both effects is given by:

$$h_{m+1} = Bx_m x_m' B' + aRV_m \tag{31}$$

where a is a non-negative parameter.

In the remainder of the paper, the series relating to industrial production, per-capita consumption and labor income will be excluded based on the results of the strictly exogenous specification. While there was some evidence of significance of these variables (or transformation) it was hardly compelling. Table

5 contains results for the six variance series when the 5 remaining core variables are included in model with ARCH effects. The parameter on the lagged realized volatility is highly significant in all of the models. More importantly, the inclusion of ARCH effects renders all but five of the explanatory variables insignificant at the 10% level. The default spread for the BH portfolio and the term premium for the SH portfolio remain significant, while unexpected inflation exhibits the most significance, with p-values between 2% and 12% for the six portfolio variances. However, joint tests of significance continue to indicate that the effect of exogenous regressors is statistically significant for all 6 portfolios at the 10% level and 4 of the 6 are below 1%. In addition, the signs of the APE are fairly stable, although the magnitude has generally diminished.

[INSERT TABLE 5 HERE]

Also reported in the table are the parameters on the lagged realized volatility (ARCH) and the parameters of lagged realized volatility when no exogenous variables (ARCH (No Exog.)) are included from the following model:

$$h_{m+1} = b_1^2 + aRV_m \tag{32}$$

Comparing the two sets of parameter estimates for lagged realized volatility, the parameters markedly decrease when the explanatory variables are included. The largest change came in the two models where the parameters on lagged realized volatility were greater than one (BL and BM portfolios). The parameter on the SH portfolio also deceased considerably. The only coefficient which did not show a large decrease was in the SL portfolio, decreasing from .844 to .834. This is the series where the explanatory variables appear to have the least impact based on the p-value of the test that all of the explanatory variables were zero.

[INSERT TABLE 6 HERE]

Table 6 contains results of models which include the 11 variables in the expanded set and ARCH effects. With the inclusion of additional explanatory variables, the effect of unexpected inflation is now significant for all of the 6 variance series at the 10% level. Variables based on the term premium are significant for multiple series using a 10% test, some are marginally significant among a number of series. The default premium is significant in 2 of the 6 series using a 20% test. Also, the short rate and the one year change in the term premium show marginal significance in multiple series. While many of the parameters were individually insignificant using standard test sizes, joint tests show that all of the models reject the null that the parameters on the explanatory variables are zero with o-values below 1%. Finally, examining the coefficients on ARCH terms, the inclusion of the expanded set of explanatory variables has resulted in a statistically significant decrease in the effect of lagged realized variance, although they remain highly significant. The average decrease in the ARCH parameter is .208 across the six series while persistence of shocks has decreased from near infinite for some of the models to at most 4 months.

Figure 7 contains a plot of the strictly exogenous model with the 8 core explanatory variables and the augmented ARCH specification with the remaining 5 of the 8 core variables (%ΔIP and %ΔPCC were excluded based on preliminary results). There are large and persistent differences between the fit of the two models, especially for the small portfolios. However, the exogenous model is able to explain many of the long swings in the variance of these six portfolios. Figure 8 contains a similar plot with the strictly exogenous model and the augmented model, both using the expanded set of explanatory variables. The fit of these two series is considerable closer than was that fit between the smaller exogenous model was to its augmented counterpart. The exogenous model with the expanded set of explanatory variables also appears to produce a closer fit to its augmented counterpart for the Large portfolios. Figure 9 contains a plot of the ratio of the fit variance from a model with only ARCH effects to a model with ARCH effects and the expanded set of explanatory variables. In most of the series, persistent deviations can be seen. In the small market equity portfolios, deviations (from 1) can be seen in the mid 1970s, early 1980s and early 1990s. For the large market equity portfolios, persistent deviations can be seen during the early to mid 1980s. Further, while the augmented model is higher than the simple ARCH specification in the small market equity portfolios in the early 1980s, it is smaller for the large market equity portfolios during this period.

[INSERT FIGURES 7,8,9 HERE]

While the log-likelihood of each model is a goodness of fit criteria, it is not directly interpretable. To better understand how well (or poorly) the different models 'fit' the data, a simple regression was run on the realized variances and the fit variances from these models. The regression fit was:

$$RV_m = \alpha + \beta \hat{h}_m + \epsilon_m \tag{33}$$

Table 7 contains results of these regressions for the 5 models. The core set of explanatory variables is only able to capture 2 to 4% of the variation in the realized variance while the expanded set is able to explain between 2 and 8%. The simple ARCH specification generally produces a better fit. However, the augmented ARCH models, particularly the model with the expanded set of explanatory variables is capable of producing a 40% better fit than the model with only lagged realized variance. Including a dummy for October 1987 dramatically increases the fit for all of the series. As previously discussed, October 1987 is responsible for more than half of the variance of the realized variances. The exogenous models appear much more capable in this panel and are able to produce good fits. Most of this is due to the ability of the models with exogenous variables to pick up low frequency patterns in the realized volatilities.

[INSERT TABLE 7 HERE]

The monotone decreases in the ARCH parameters add more credibility to the exogenous variables than the significance of their coefficients. Fit volatilities from the exogenous models are generally slowly varying (in general, too slowly varying) processes. However, the inclusion of a slowing changing series into the ARCH has dramatically lowered persistence. These two empirical findings point to the idea that there are multiple time scales in the variance of returns, a high frequency component with low persistence and a low frequency component with a business cycle length level of persistence. Recently, Engle & Lee (1999) have constructed a model using only lagged squares which allows for two time-scales in variances. The findings of this paper are consistent with this model despite the different methodology.

Robustness Checks

A number of robustness checks were performed to examine the effect of a particular period or methodology.[10] A number of alterations to the model presented were considered, including:

- Changes in the dependant variable
- Changes in the mean model

[10] To conserve space, numerical results from these models are not presented with the paper. They are available upon request.

- Subperiod analysis
- Different lag specifications for the arch effects.

Section 2 discussed an alternative estimator of the realized covariance using a Newey-West style estimator with one lag to capture auto- and cross-correlations. The realized volatility under this new estimator becomes

$$RV_m = \sum_{d=1}^{DM} r_{md} r'_{md} + \frac{1}{2}(r_{md} r'_{md-1} + r_{md-1} r'_{md}) \tag{34}$$

The results of all four models were repeated using this estimator of the realized covariance in place of the simple realized covariance estimator. The results using this estimator in-place of the simpler estimator are essentially unchanged. The signs of the APEs are identical as are the magnitudes. The APEs typically are within 10% of those estimated using the simple RV estimator. If there is any difference, it is that there is slightly more evidence of significance using the Newey-West style estimator than the simple estimator, and the only variables to show a decrease in significance are the 3 core macroeconomic variables, industrial production, consumption, and labor income (and their transformation). Further, in the models augmented with ARCH, the decrease on persistence is slightly larger using the Newey-West style realized covariance estimator.

In addition to the Newey-West estimator, monthly close-to-close prices were used in place of the realized volatility. In this case the model becomes and exact ARCH model. The primary difference found using close-to-close was that there was virtually no evidence of ARCH effects found in the data. Using an ARCH-LM test, 5 of the 6 series fail to eject the null of no ARCH at the 10% level. Further, in models with only ARCH effects (no exogenous variables), the estimated ARCH coefficient was exactly 0 in 3. However, there was widespread evidence of statistically significant exogenous variables, all with the same sign and similar magnitude as those estimated using the realized variance estimator.[11]

As an alternative to estimating no conditional mean, 3 specifications for the conditional mean were considered: subtracting the risk-free rate observed during month $m-1$ were subtracted from returns in month m, an AR(1) on monthly returns to estimate the conditional mean, and an OLS specification where the regressors were the core set of 8 explanatory variables. In all four models estimated, none of the 3 mean treatments produced a single change in significance of an individual coefficient and the log-likelihood change was less than one point in all models. This was not a particularly surprising result given the structure of the realized volatility estimator and the small amount of predictability in monthly returns, where the largest R^2 of any of the models was less than 3%.

Three sub-samples were also examined. The first was to examine the full sample without October 1987. The second sub-period considered examined the sample from 1963 until the end of 1996 in order to examine whether the recent increase in volatility was driving the results (Campbell et al. 2001). Finally, the third was to examine the effect to the high inflation period ending in 1982. Unlike two previous changes, there are some interesting changes in terms of parameter significance under one of these treatments, although the fundamental conclusions are unchanged.

In the treatment where October 1987 was dummied out in the model with the 8 core explanatory variables, none of the coefficients on industrial production, labor income or per-capita consumption growth

[11] The quality of the volatility measurement using monthly realized volatilities was so dramatically different from that using monthly close-to-close returns that there was no requirement for the AR(∞) implied by the inclusion of a smoothing (GARCH) term.

were significant.[12] This result contrasts with the original findings were there was some evidence of statistical significance for industrial production growth. This trend continued in the expanded model where fewer of these three variables and their transformations were significant. However, the largest changes came in the models with ARCH effects. In the model with 5 explanatory variables and ARCH effects many of the individual coefficients are significant, primarily the term premium and the default spread. This stands in stark contrast to the results when October 1987 was included and little individual significance was found despite widespread evidence of joint significance. Similar results were found in the expanded model with ARCH effects. Finally, while the inclusion of explanatory variables lowered the ARCH coefficient in all models, they were uniformly lower than in the case when October 1987 was included in the sample. In both of the other sub-sample treatments, the strong evidence of significance of the exogenous variables remained.

Finally, to examine whether the macroeconomic variables represent a unique source of information or are merely a proxy for omitted lags, specifications with 2 and 3 lags of realized covariance were examined. While the a second lag could not be rejected in 4 of the six variance series, the third could be rejected in all 6 and the statistical significance of the exogenous variables was unaffected. Further, the persistence measured as the sum of the coefficients is essentially identical. These robustness checks provide strong evidence that the macroeconomic variables contain a different type of information than lagged returns.

Quarterly

It is possible that monthly covariances are too high-frequency to capture macroeconomic phenomena, particularly aggregates suchs as consumption or labor-income growth. As an alternative to the use of monthly realized covariances, quarterly realized covariances can be considered with in the framework of this model. The same four models were fit to quarterly data as were fit to monthly data. In an effort to conserve space, the results are contained in a separate appendix available upon request.

The results using quarterly realized volatilities and quarterly data are very similar to those when monthly were used. In the strictly exogenous model with the 8 core explanatory variables, wide spread significance was found for 4 of these variables. The APE of the growth rate of the CLI was negative and significant at the 10% level in all 6 quarterly variance series. The effect of the default premium was significant in 5 out of the 6 series (all but SL) and had a positive effect in all 6 series. Both the short term rate and the term premium had negative average partial effects and were significant in all 3 of the small firm portfolios and the short term rate was also significant for the BH portfolio. The growth rate of industrial production and of per-capita consumption was never significant while the growth rate of labor income was only significant for the SH portfolio. In the expanded model, similar results to the monthly models were found with one notable difference. The one year growth rate of per-capita consumption was significant in 5 of the 6 series (and had a p-value of 12% in the 6^{th}). However, the sign of the APE in all 6 models was positive, a somewhat perplexing result. One possible explanation is that consumption is actually lagging market volatility and the quarterly data is allowing the model to pick an alternative cycle.

When ARCH effects were included with the 5 core variables, the models were generally not able to reject the null that the effect of the exogenous variable was zero at conventional sizes. However, the p-value was less than 25% in 4 of the 6 series. The difference here can be attributed to the lack of data affecting the precision of the estimated as the APE signs were consistent with the no-ARCH models. Finally, in the expanded model with 11 explanatory variables, the null of no effect of exogenous variables can be rejected at the 10%

[12]The estimation was conduced by multiplying the log-likelihood by a dummy variable which took the value 0 in October 1987 and was 1 elsewhere. Standard errors reflect the omission of October 1987 as well.

level for all 6 series. Further, there was evidence of a significant effect for growth rate of the CLI in 3 of the 6 series with another marginally significant. Unexpected inflation is also significant in 3 or the 6 series. The next section will discuss the effect of exogenous variables on the covariance and correlation of returns.

5 Covariance and Correlation Results

This section will build on the previous findings for conditional variances by examining the explanatory properties of exogenous variables for the covariance of the six portfolio returns. Four models will be examined which correspond to the univariate specifications previously explored: two strictly exogenous models, one with the core set of 5 variables (the three which were rarely significant for the univariate specifications are not considered) and one with the expanded set of 11 explanatory variables and two models which include ARCH effects in addition to the explanatory variables. The realized covariance of the six portfolios is modeled according to section 2 using daily returns to construct monthly covariance:

$$RV_m = \sum_{d=1}^{D_m} r_{md} r'_{md} \tag{35}$$

The strictly exogenous model remains the same as used in the univariate section

$$H_{m+1} = B(I_k \otimes x_m x'_m)B' \tag{36}$$

and the specification with ARCH effects are included will be modeled using a parameterization which will ensure that parameter matrix on the lagged realized covariance will be positive semi-definite:

$$H_{m+1} = B(I_k \otimes x_m x'_m)B' + AA' \odot RV_m \tag{37}$$

where A is a symmetric matrix analog of B.[13] A multivariate ARCH specification without exogenous variables will also be examined. This specification is nested in the exogenous specification when $x_m = 1$. In this specification, the conditional covariance is given by:

$$H_{m+1} = BB' + AA' \odot RV_m \tag{38}$$

In an analogous fashion to the use of average partial effects in the examination of the variance results, the average effect of an explanatory variable on the conditional correlation will be used throughout this section. Specifically, the tables contain

$$\overline{PE}_{ij} = \frac{1}{M}\sum_{m=1}^{M} \frac{\partial R_{ijm}}{\partial x_m} = \frac{1}{M}\sum_{m=1}^{M} \frac{\partial \frac{H_{ijm}}{\sqrt{H_{iim}H_{jjm}}}}{\partial x_m} = \tag{39}$$

$$\frac{1}{M}\sum_{m=1}^{M}\left(\frac{\partial H_{ijm}}{\partial x_m} H_{iim}^{-\frac{1}{2}} H_{jjm}{}^{-\frac{1}{2}} - \frac{1}{2}\frac{\partial H_{iim}}{\partial x_m} H_{ijm} H_{iim}^{-\frac{3}{2}} H_{jjm}{}^{-\frac{1}{2}} \frac{1}{2}\frac{\partial H_{jjm}}{\partial x_m} H_{ijm} H_{iim}^{-\frac{1}{2}} H_{jjm}{}^{-\frac{3}{2}}\right)$$

Appendix B details the derivative of H_m.

[13]The specification used is the same as discussed in section 2 where the ARCH parameters were allowed to depend on exogenous variables. However, the only explanatory variable included in the innovation matrix is a constant. In this case $A(I_k \otimes x_m x'_m)A' = A(I_k \otimes 1)A' = AI_kA' = AA'$.

Table 8 contains joint parameter significance results for the four models with exogenous variables. The top panel of the table contains test results that all of the parameters on the explanatory variables are zero. In all four models the null is strongly rejected with a p-value of 0. Also listed is the number of degrees of freedom for each test, given by $NK(K+1)/2$ where N is the number of explanatory variables (not including a constant) in the model. The test statistics were larger when no ARCH effects were included in the model, although the parameters were still highly significant even in the models with ARCH.

[INSERT TABLE 8 HERE]

The bottom panel in table 8 contains test statistics and p-values for significance of the specific explanatory variables. The null hypothesis in each of the tests is

$$H_0 : b_{11n} = b_{12n} = b_{1Kn} = b_{22n} = \ldots = b_{KKn} = 0$$

where n indexes the position of the variable being tested and will be asymptotically distributed $\chi^2_{K(K+1)/2}$. In all of the tests discussed here, $K(K+1)/2 = 21$. In the strictly exogenous model with the core set of 5 explanatory variables (labeled (1)), all of the explanatory variables are statistically significant at the 10% level. Column 2 contains significance results for the 5 variable model when ARCH effects are included. 4 of the 5 variables are significant while unexpected inflation has moved from marginally significant to insignificant. Moreover, the significance level of each of the variables is lower when ARCH effects are included than in a model with only exogenous factors. However, the significance results when ARCH effects are included are considerably different than what was found in the univariate specifications where the inclusion of lagged realized variances rendered most parameters individually insignificant at conventional levels.

Columns 3 and 4 contain results for the expanded set of 11 explanatory variable with (4) and without (3) ARCH effects. Ten of the 11 explanatory variables are significant at the 10% level in the model without ARCH. When ARCH effects are included, the number of significant variables decreases to 9, although two variables flip from marginally significant to insignificant and vice versa. However, the number of statistically significant explanatory variables is considerably higher in the multivariate specification than was the case in the univariate specifications. Further, variables which were high persistence, such as the default premium, term premium, and short rate, remain significant. The increased significance levels of the exogenous variables indicate that the explanatory variables are better at explaining correlation changes than lagged correlation (or variance and covariance).

However, there is a subtle but important difference in the structure on the lagged realized covariances in the multivariate models. In all six of the univariate specifications without exogenous variables, the parameter on lagged realized covariance was high, generally close to 1, indicating high persistence in volatility. However, in the multivariate ARCH specification, even in a model without exogenous variables, the parameters on the lagged realized covariances were typically below 0.7 and always below .8. Table 9 contains the estimated coefficients on the lagged realized covariance for the three models where it was included. The parameters on the diagonal of each matrix correspond to variance innovations while the off diagonal correspond to the loadings on lagged realized covariance. The loadings on lagged realized variance decrease uniformly as the number of exogenous variables is increased from 0 to 5 to 11.

[INSERT TABLE 9 HERE]

The diagonal elements of the specification without exogenous variables were between .56 and .75 and are markedly different than their corresponding values when considered in the univariate specification, where they ranged from .79 to 1.01.[14] Including the core set of explanatory variables decreased the diagonal elements .06 on average with a range of .01 to .09. Including the expanded set of 12 explanatory variables resulted in an average decrease of .14 with a range from .10 to .18. These large decreases where the parameters on the lagged realized variance were typically 20% smaller than those without explanatory variables provide further evidence of two properties of covariances: multiple time scales and significant effects of lagged exogenous variables.

[INSERT TABLE 10 HERE]

While the emphasis of the section is on the conditional correlations and not the variances, the APEs (equation 30) were calculated for the four models and are presented in table 10. The sign on the vast majority of the APEs agree with the findings of the univariate specifications although there is more uniformity in the levels. In addition, there is more evidence of a statistically significant effect of the explanatory variables in the full specification, particularly when ARCH effects are included. However, the results of the conditional variance series are similar to the exposition in section 4 to not warrant further discussion.

[INSERT TABLE 11 HERE]

[INSERT TABLE 12 HERE]

Tables 11 and 12 contain the average partial effect on correlation in the models using the core set of variables without and with ARCH effects, respectively. While is not possible to determine whether the effect of an exogenous variable has significant effect on the conditional correlation, it is possible to determine if the variable is a significant determinant of conditional covariance. In these tables, bold indicates that the effect of the variable on the conditional covariance between two assets was statistically significant at the 10% level. In the model without ARCH effects, there is overwhelming evidence of a statistically meaningful relationship between the core set of explanatory variables and the conditional covariances. All save 3 of the 75 conditional covariance relationships were significant. There are also some interesting patterns evident in the APEs. The growth rate in the composite leading indicator is negative for all pairs. This indicated than a decrease in the CLI would correspond to higher return correlation, particularly between small and large firms. The default premium has a positive effect for each pair, indicating that large default premiums correspond to more comovement in prices. The effect of the short term interest rate is mixed but is the generally negative between small and large firms, indicating that returns of these two classes decouple when short term rates are high. One possible explanation for this is the asymmetric exposure to credit markets that small and large firms face. The term premium generally has a negative effect while the effect of unexpected inflation is positive for all but two of the pairs.

When the model augmented with ARCH is considered (table 12), the effects diminish although there still is widespread evidence of statistical significance. Further, there is almost 100% sign agreement between the APES in the two tables and the magnitudes are similar. This adds credence to the notion that ARCH effects are successful at capturing volatility clustering but that state variables are possibly better at capturing changes in the comovements of returns.

[INSERT TABLE 13 HERE]

[14] These findings were robust to the choice of starting values for the AA' matrix. A number of different starting values for AA' were used, ranging from near 0 to near 1 and produced identical results to 3 decimal places.

[INSERT TABLE 14 HERE]

In an analogous manner to the two previous tables, tables 13 and 14 contain results from the expanded model without and with ARCH effects. A number of the extended set of variables evidence significance in these models. Specifically, monthly changes in the default premium and the term premium are widely significant as are one year changes in the composite leading indicator. However, the average effect of many of these variables is near zero, indicating that they can have effects of both signs. Further, the magnitude of the APE on the change variables (either yearly or monthly) is generally lower than on the levels variables. Finally, the effect of including the ARCH term is identical to the smaller model: little changes in either the magnitude and the statistical significance is found.

Figures 10, 11, and 12 each contain plots of the fit correlation and the realized correlation from four of the five multivariate models described here. Figure 10 contains the plots of the correlation between the Small-Low portfolio and the Small-High portfolio. The four plots contain the fit from the strictly exogenous specification using the 5 core variables (upper-left), the strictly exogenous specification with the expanded set of 12 variables (upper-right), the ARCH specification with no exogenous variables (lower-left), and the model with both the expanded set of explanatory variables and ARCH effects (lower-right). The model with only the core set of explanatory variables shows virtually no dynamics. However, the fit with expanded set is similar to the fit produced by either the ARCH model or the expanded set and ARCH model. Figures 11 and 12 show similar patterns: an inability of the core variables model to capture all of the dynamics while the fit of the other three appear similar, although neither of the exogenous models appear capable of explaining the large changes in correlation which occurred during the stock market run-up of the later 1990s.

Finally, table 17 contains regression results from a regression of realized variance (realized correlation) on fit variance (correlation). The regression fit was:

$$RV_m^{ii} = \alpha + \beta \hat{H}_m^{ii} + \epsilon_m \tag{40}$$

for the variance series, where $\hat{H}_m^{ii}$ is the fit variance from one of the five models, or

$$RC_m^{ij} = \alpha + \beta \hat{R}_m^{ij} + \epsilon_m \tag{41}$$

for the variance series, where $\hat{R}_m^{ii}$ is the fit correlation from one of the five models. The ARCH models , either with or without exogenous variables, are considerably better at fitting the variance series than the either of the models with only exogenous variables. However, the results for correlation show that even the smaller exogenous model is capable of better explaining changes in correlation than the ARCH model. The larger model with only exogenous variables produces better fit in 11 of the 15 correlation series than the simple ARCH specification. These results where the exogenous variables are better able to explain correlation than an ARCH specification are consistent with the idea that correlations are slowly evolving series and that ARCH models are two responsive to short term news.

Robustness Checks

One issue with estimating the full multivariate model is there are a large number of parameters relative to the sample size in some of the specifications. As a robustness check to the findings of the full multivariate model, two experiments were performed. First, bivariate models with the same explanatory variables were fit to the 21 pairs of assets. Second, a model was fit where the effect of the exogenous variables was pooled across assets.

The bivariate specifications has considerably fewer parameters than the multivariate specifications. For instance, when 5 explanatory variables were included in the model with all 6 portfolios, the model would have 126 parameters ignoring ARCH effects. However, a model between two assets would have only 18. This may allow for more precise estimation of the parameter variance. Tables 15 and 16 contain the equivalent of tables 12 and 14, respectively using 15 bivariate models in place of the full multivariate specification. While there is some evidence changes in significance, the general results are robust. In the multivariate model, the growth rate of the composite leading indicator had negative partial effects for each pair and was much stronger between small and large firm portfolios than it was within these groups. In the bivariate specifications, the significance is reduced but the sign is negative everywhere. Similarly, the default premium had, on average, a positive partial effect for all but two of the pairs in both the full multivariate specification and the bivariate specifications. The short rate typically had a negative effect in both while the effect of the term premium was mixed in both. The only substantial difference is in the statistical significance of unexpected inflation, where it is generally insignificant in the bivariate model but is highly significant in the full multivariate specification.

The extended specification had a similar pattern where there was widespread agreement on the sign of the average partial effect, although there was more stronger of statistical significance in the full specification. One possible cause is that the full specification is more efficient at estimating the parameters of the conditional covariance than the bivariate specifications. Alternatively, the loadings on lagged realized covariance were smaller in the full multivariate case than in the bivariate models, even when no exogenous variables were included. The lower information content of lagged realized covariance in the full specification left information for the explanatory variables.

Finally, a pooled specification was considered. In the pooled specification, the parameters on the constant were left unconstrained but the loadings on the explanatory variables were pooled together per variable. In other words, $b_{ijn} = b_n$ for all i, j. This specification economizes on parameters but severely restricts the allowable dynamics. In a 6 asset model, the number of parameters is reduced from 126 to 26 when there are 5 explanatory variables in addition to a constant. If the loadings on the constant were the same across all assets, this would reduce to having the same partial effect for each asset pair, a less than believable hypothesis. However, if the explanatory variables remain significant under this severe restriction, it lends credibility to the explanatory power of the exogenous variables.

In the strictly exogenous specification with the core set of 5 explanatory variables, all but unexpected inflation were significant at the 10% level, and the explanatory variables were jointly significant below the 1% level. When augmented with ARCH, the same four variable remain significant and the joint hypothesis is rejected below the 1% level. In the extended model without ARCH, all but the growth in the CLI and the short term rate are significant at the 10% level, while in the extended model with ARCH effects, four of the variables, growth in the CLI, monthly changes in the default premium, the short term rate, and one year changes in inflation, are insignificant at the 10%. While some of these were significant in the full specification, the p-values are below 15% for the growth in the CLI and the short term rate. The lack of significance may be attributable to a false pooling restriction. These robustness checks add credibility to the incremental explanatory power of the explanatory variables.

6 Conclusion

This paper develops a model for the conditional covariance of asset returns which using variables *other* than lagged cross-products to explore the dynamics of equity covariance. This paper represents a major deviation from the methodology previously employed in the literature. The main finding is that state variables, previously related to predictable components in returns, have significant explanatory power for the covariance of equity returns, even when lagged cross-products of returns are included. The paper also proposes a new method for estimation of parameters in monthly covariances models which leverages the high quality measurement of monthly covariances constructed from daily data (monthly realized covariances) in a QMLE framework.

Using the 6 Fama-French portfolios, this paper finds widespread evidence of statistically significant variation in conditional covariances attributable to financial and macroeconomic state variables. Allowing for ARCH effects diminishes, but does not remove, the explanatory power of exogenous factors. A number of variables which previously been found to predict the conditional mean of equity returns have been shown to have predictive power over the variance and covariance of equity returns. Specifically, the term premium, the default premium and the short rate were statistically significant in most models. Among the macroeconomic variables, changes in the composite leading indicator evidenced predictive power over conditional covariances, while per-capital consumption growth, labor income growth and industrial production growth had little explanatory power over conditional variances. This paper also documents a meaningful relationship between variables which represent changes in the state of the economy such as year-over-year changes in the term premium to the conditional covariance of equity returns. The statistical significance of these variables is consistent with a theory that volatility is related to uncertainty about the state of the economy and not necessarily the state of the economy.

These findings were robust to the choice of realized covariance estimator or the inclusion of a conditional mean, although sub-sample analysis did change the significance of some of explanatory the variables. Specifically, omitting October 1987 from the sample provided diminished the significance of unexpected inflation, although the results were robust to excluding the recent high volatility period beginning in 1995. Further, the same effects were evidenced using quarterly data in addition to monthly.

A number of interesting questions remain. For instance, does the inclusion of exogenous variables improve the asset allocation performance of the models. Standard ARCH models have notoriously poor performance when used to construct minimum-variance portfolios, likely due to over-reaction to recent news about the level of volatility. However, the decreased magnitude of the ARCH parameters in these models may provide a meaningful improvement over models which rely exclusively on lagged cross-products of returns. Another interesting, but unanswered question is what is driving the apparent multiple time-scales in covariance. A number of theoretical models are capable of generating time-varying volatility. However, in general, these only generate a single volatility cycle with a short period and exponential decay. This contrasts with the findings of this papers where a long period was evidenced typically in excess of 6 year. Finally, if there are two cycles in volatility, these may affect mean-variance regressions in in an unpredictable manner. Numerous investigations between the mean and variance of asset returns have produced conflicting results. Some find that the relationship is positive, others negative, and yet others find it is not different from zero. However, examination of multiple time-scales in conditional variances may provide insight into these discrepancies. I leave these as topics for further research.

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Appendix A

This appendix will consider the choice of the square-root used in modeling the conditional covariance of returns. Consider a model with two assets (r_1 and r_2) and 2 explanatory variables (x_1 and x_2).

$$\begin{bmatrix} r_{1t+1} \\ r_{2t+1} \end{bmatrix} |\mathcal{F}_t \sim N(0, H_{t+1}) \tag{42}$$

A model employing the Cholesky factor linear in the explanatory variables would be of the form:

$$H_{t+1}^{1/2} = B(I_k \otimes x_t) = \begin{bmatrix} b_{11}x_t & 0 \\ b_{12}x_t & b_{22}x_t \end{bmatrix} = \begin{bmatrix} b_{111}x_{1t} & b_{112}x_{2t} & 0 & 0 \\ b_{121}x_{1t} & b_{122}x_{2t} & b_{221}x_{1t} & b_{222}x_{2t} \end{bmatrix} \tag{43}$$

and the conditional covariance would be

$$\begin{array}{rcl} H_{t+1}(1,1) & = & b_{111}^2 x_{1t}^2 + 2b_{111}b_{112}x_{1t}x_{2t} + b_{112}^2 x_{2t}^2 \\ H_{t+1}(1,2) & = & b_{111}b_{121}x_{1t}^2 + (b_{111}b_{122} + b_{112}b_{121})x_{1t}x_{2t} + b_{112}b_{122}x_{2t}^2 \\ H_{t+1}(2,2) & = & (b_{121}^2 + b_{221}^2)x_{1t}^2 + 2(b_{121}b_{122} + b_{221}b_{222})x_{1t}x_{2t} + (b_{122}^2 + b_{222}^2)x_{2t}^2 \end{array} \tag{44}$$

Considering the model with the asset order switched, using C to denote the parameters in this case. C should a function of B, and vice versa. we find:

$$\begin{bmatrix} r_{2t+1} \\ r_{1t+1} \end{bmatrix} |\mathcal{F}_t \sim N(0, \tilde{H}_{t+1}) \tag{45}$$

$$\tilde{H}_{t+1}^{1/2} = C(I_k \otimes x_t) = \begin{bmatrix} c_{11}x_t & 0 \\ c_{12}x_t & c_{22}x_t \end{bmatrix} = \begin{bmatrix} c_{111}x_{1t} & c_{112}x_{2t} & 0 & 0 \\ b_{121}x_{1t} & c_{122}x_{2t} & c_{221}x_{1t} & c_{222}x_{2t} \end{bmatrix} \tag{46}$$

and the conditional covariance would be

$$\begin{array}{rcl} \tilde{H}_{t+1}(1,1) & = & c_{111}^2 x_{1t}^2 + 2c_{111}c_{112}x_{1t}x_{2t} + c_{112}^2 x_{2t}^2 \\ \tilde{H}_{t+1}(1,2) & = & c_{111}c_{121}x_{1t}^2 + (c_{111}c_{122} + c_{112}c_{121})x_{1t}x_{2t} + c_{112}c_{122}x_{2t}^2 \\ \tilde{H}_{t+1}(2,2) & = & (c_{121}^2 + c_{221}^2)x_{1t}^2 + 2(c_{121}c_{122} + c_{221}c_{222})x_{1t}x_{2t} + (c_{122}^2 + c_{222}^2)x_{2t}^2 \end{array} \tag{47}$$

Logical consistency would require that $H_{t+1}(1,1) = \tilde{H}_{t+1}(2,2)$, $H_{t+1}(1,2) = \tilde{H}_{t+1}(1,2)$ and $H_{t+1}(2,2) = \tilde{H}_{t+1}(1,1)$ which provides the 9 following equivalences:

$$\begin{array}{lrcl} (1) & b_{111}^2 & = & c_{121}^2 + c_{221}^2 \\ (2) & b_{111}b_{112} & = & c_{121}c_{122} + c_{221}c_{222} \\ (3) & b_{112}^2 & = & c_{122}^2 + c_{222}^2 \\ (4) & b_{111}b_{121} & = & c_{111}c_{121} \\ (5) & b_{111}b_{122} + b_{112}b_{121} & = & c_{111}c_{122} + c_{112}c_{121} \\ (6) & b_{112}b_{122} & = & c_{112}c_{122} \\ (7) & b_{121}^2 + b_{221}^2 & = & c_{111}^2 \\ (8) & b_{121}b_{122} + b_{221}b_{222} & = & c_{111}c_{112} \\ (9) & b_{122}^2 + b_{222}^2 & = & c_{112}^2 \end{array} \tag{48}$$

Solving for the elements of C in terms of those of B using equivalences $\{1,3,4,6,7,9\}$ above, the following set of equivalences arise:

$$\begin{array}{rcl} c_{111} & = & \sqrt{b_{121}^2 + b_{221}^2} \\ c_{112} & = & \sqrt{b_{122}^2 + b_{222}^2} \\ c_{121} & = & \frac{b_{111}b_{121}}{\sqrt{b_{121}^2+b_{221}^2}} \\ c_{122} & = & \frac{b_{112}b_{122}}{\sqrt{b_{122}^2+b_{222}^2}} \\ c_{221} & = & \sqrt{b_{111}^2 - \frac{b_{111}^2 b_{121}^2}{b_{121}^2+b_{221}^2}} \\ c_{222} & = & \sqrt{b_{112}^2 - \frac{b_{112}^2 b_{122}^2}{b_{122}^2+b_{222}^2}} \end{array} \tag{49}$$

Consider one parameterization for B:

$$B = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 1 & -1 & 1 & -1 \end{bmatrix} \tag{50}$$

Using the equivalences above C can be written as:

$$C = \begin{bmatrix} \sqrt{2} & \sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -1/\sqrt{2} & 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix} \tag{51}$$

While only 6 of the 9 equations were used in deriving the equivalence of B and C, all 9 must hold for there to be an identical relationship between the two. Consider the 8^{th} relationship between B and C: $b_{121}b_{122} + b_{221}b_{222} = c_{111}c_{112}$. Substituting in the values for B and C, this expression evaluates to $1 \cdot -1 + 1 \cdot -1 \neq \sqrt{2} \cdot \sqrt{2}$, demonstrating that there are parameterizations when $[r_{1t}, r_{2t}]'$ which cannot be equivocated any parameterizations for $[r_{2t}, r_{1t}]'$.

The spectral square-root, on the other hand, makes finding n equivalent parameterizations simple. block symmetry of the parameter matrix allows for the switching of rows and columns of *blocks* of the parameter matrix when the order of returns is switched. Returning to the problem being discussed, the three elements of H_{t+1} can be written (using block forms for the parameters, see equation 6):

$$\begin{array}{rcl} H_{t+1}(1,1) & = & b_{11}x_t'x_t b_{11}' + b_{12}x_t'x_t b_{12}' \\ H_{t+1}(1,2) & = & b_{11}x_t'x_t b_{12}' + b_{12}x_t'x_t b_{22}' \\ H_{t+1}(2,2) & = & b_{12}x_t'x_t b_{12}' + b_{22}x_t'x_t b_{22}' \end{array} \tag{52}$$

Finding an equivalent parameterization for $[r_{2t}\, r_{1t}]$ involves switching the rows and columns of B.

$$C = \begin{bmatrix} c_{11} & c_{12} \\ c_{12} & c_{22} \end{bmatrix} = \begin{bmatrix} b_{22} & b_{12} \\ b_{12} & b_{22} \end{bmatrix} \tag{53}$$

and

$$\begin{array}{rcccl} \tilde{H}_{t+1}(1,1) & = & c_{11}x_t'x_t c_{11}' + c_{12}x_t'x_t c_{12}' & = & H_{t+1}(2,2) \\ \tilde{H}_{t+1}(1,2) & = & c_{11}x_t'x_t c_{12}' + c_{12}x_t'x_t c_{22}' & = & H_{t+1}(1,2) \\ \tilde{H}_{t+1}(2,2) & = & c_{12}x_t'x_t c_{12}' + c_{22}x_t'x_t c_{22}' & = & H_{t+1}(1,1) \end{array} \tag{54}$$

Appendix B

This section will describe the calculation of various derivatives necessary for the estimation of the parameter variance and for the calculation of the average partial effects. The general model specification for the conditional covariance in month m is given by:

$$H_{m+1} = B(I_k \otimes x_m)(I_k \otimes x_m)'B' + \sum_{q=1}^{Q} A_q A_q \odot RV_{m-q+1} \tag{55}$$

Let θ denote the unique parameter of B and $A_1, \ldots, A_Q$. The log likelihood for month m is given by:

$$LL_m(r_m, \theta) = -\frac{1}{2} D_m \left(k \ln(2\pi) + \ln(|\bar{H}_m|) + \text{tr}(\bar{H}_m^{-1} \frac{RV_m}{D_m}) \right) \tag{56}$$

The score of the log-likelihood with respect to θ_i is given by

$$\frac{\partial LL_m}{\partial \theta_i} = -\frac{D_m}{2} \text{tr}(\frac{\partial \bar{H}_m}{\partial \theta_i} \bar{H}_m^{-1} - \frac{RV_m}{D_m} \bar{H}_m^{-1} \frac{\partial \bar{H}_m}{\partial \theta_i} \bar{H}_m^{-1}) \tag{57}$$

making use that $\ln(|\bar{H}_m|) = \text{tr}(\ln(|\bar{H}_m|))$, $\frac{\partial \ln(|\bar{H}_m|)}{\partial \theta_i} = \frac{\bar{H}_m^{+}}{|\bar{H}_m|} = \bar{H}_m^{-1}$ and that $\frac{\partial \text{tr}(\frac{RV_m}{D_m} \bar{H}_m^{-1})}{\partial \theta_i} = \text{tr}(\frac{RV_m}{D_m} \frac{\partial \bar{H}_m^{-1}}{\partial \theta_i})$.

The asymptotic variance is calculated using a Newey-West estimator,

$$\hat{\Sigma} = \frac{1}{M} \sum_{m=1}^{M} \left(s_m s_m' + \sum_{l=1}^{L} w_i (s_m s_{m-l}' + s_{m-l} s_m') \right) \tag{58}$$

where $s_m = \frac{\partial \bar{H}_m}{\partial \theta}$, l is the maximum lag and $w_i = \frac{l+1-i}{l+1}$. The hessian can be estimated using

$$\frac{\partial LL_m}{\partial \theta_i \partial \theta_j} = -\frac{D_m}{2} \text{tr}(\frac{\partial \bar{H}_m}{\partial \theta_i} \bar{H}_m^{-1} \frac{\partial \bar{H}_m}{\partial \theta_j} \bar{H}_m^{-1}) \tag{59}$$

The estimate of the expected value of the hessian is calculated using the sample analog to the expectation:

$$\hat{\Gamma}_{ij} = \frac{1}{M} \sum_{m=1}^{M} \frac{\partial LL_m}{\partial \theta_i \partial \theta_j} \tag{60}$$

The partial effect of θ_i on H_m is given by

$$\frac{\partial H_m}{\partial b_{ijn}} = \frac{\partial B}{\partial b_{ijn}} (I_k \otimes x_m x_m') B' + B(I_k \otimes x_m x_m') \frac{\partial B'}{\partial b_{ijn}} \tag{61}$$

if θ_i is in B and is given by

$$\frac{\partial H_m}{\partial a_{ijq}} = \left(\frac{\partial A}{\partial a_{ijq}} A_q + A_q \frac{\partial A}{\partial a_{ijq}} \right) \odot RV_{m-q} \tag{62}$$

if θ_i is in A_q.

Finally, the calculation of the average partial effects is straight forward. The effect of the exogenous variables is completely determined by the intercept, so ARCH terms can be safely ignored. Thus, the partial effect of x_{im}, i denoting the position of the relevant explanatory variable, on H_{m+1} is given by:

$$\frac{\partial H_{m+1}}{\partial x_m} = B(I_k \otimes e_i)(I_k \otimes x_m')B' + B(I_k \otimes x_m)(I_k \otimes e_i')B' = B(I_k \otimes e_i x_m')B' + B(I_k \otimes e_i x_m')B' \tag{63}$$

where e_i is a 0-vector with a 1 in the i$^{\text{th}}$ position.

Descriptive Statistics

	mean	std. dev	skew	kurt	skew (m)	kurt (m)
SL	0.086	0.166	-0.725	12.453	-0.361	4.726
SM	0.140	0.118	-0.951	15.095	-0.535	6.495
SH	0.159	0.114	-0.986	16.681	-0.307	7.595
BL	0.104	0.161	-0.496	17.680	-0.263	4.767
BM	0.112	0.134	-1.160	32.566	-0.258	5.278
BH	0.134	0.134	-0.943	25.794	-0.151	4.892

Annual Standard Deviation and Correlation

	SL	SM	SH	BL	BM	BH
SL	0.166					
SM	0.945	0.119				
SH	0.905	0.958	0.114			
BL	0.812	0.788	0.757	0.161		
BM	0.767	0.795	0.784	0.896	0.134	
BH	0.731	0.780	0.796	0.830	0.900	0.134

Table 1: Summary statistics for the six portfolios. The top panel contains annualized mean and standard deviation in addition to skewness and kurtosis from daily data and from monthly data (denoted (m)). The bottom panel contains annualized standard deviation on the diagonal and correlation in the off diagonal elements. Both the standard deviation and correlation were constructed using the mean of the monthly realized covariances $\left(Corr_{ij} = \frac{\sum_{m=1}^{M} RV_m^{ij}}{\sqrt{\sum_{m=1}^{M} RV_m^{ii} \sum_{m=1}^{M} RV_m^{jj}}} \right)$.

Variable Definitions
Constants

Variable	Description
CONST	A vector of ones used in the exogenous portion of the time-varying covariance.
ARCH	A vector of ones used in the ARCH potion of the time-varying covariance.

Core Variables

Variable	Description
%ΔCLI	Log-difference of The Conference Board's composite leading indicator.
DEF	The difference between the yield on seasoned corporate bonds rated BAA and seasoned corporate bonds rated AAA.
%ΔIP	Log-difference of the FRB's Industrial Production Index.
%ΔLBR	Log-difference of per capita labor income.
%ΔPCC	Log-difference of per-capita non-durable consumption.
TERM	Difference between the yield on a 10 year and a 1 year treasury.
SHORT	Yield on a 3 month treasury.
UINF	Monthly inflation minus the average inflation the previous 12 months.

Expanded Variables

Variable	Description
CLI_{1YR}	Year over year change of the log of the CLI.
DEF_{1YR}	The one year change in the default rate.
ΔDEF	The monthly change in the default rate.
INF_{1YR}	The one year log difference of CPI.
IP_{1YR}	The one year log difference of the IP index.
LBR_{1YR}	The one year growth in the labor income.
PCC_{1YR}	The one year log difference of per-capita consumption.
$TERM_{1YR}$	The one year change in the term premium.
ΔTERM	The month over month change in the term premium.

Table 2: List of variables used throughout the paper which can be divided into three subgroups. The top panel refers to the names given to the constants. The second panel contains the core set of 8 explanatory variables and the third panel contains the list of the expanded set of variables which are transformations on the core variables.

Univariate Models - Core Variables

	SL	SM	SH	BL	BM	BH
CONST	**4.279**	**4.418**	**4.581**	**3.046**	**2.755**	**3.401**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
%Δ CLI	**-0.352**	-0.309	-0.206	**-0.681**	**-0.564**	**-0.537**
	(0.094)	(0.135)	(0.303)	(0.000)	(0.003)	(0.002)
DEF	**0.908**	**1.484**	**1.609**	**1.294**	**1.433**	**1.525**
	(0.004)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
%Δ IP	**-0.242**	**-0.257**	**-0.257**	-0.099	0.009	-0.111
	(0.057)	(0.043)	(0.038)	(0.488)	(0.957)	(0.280)
%Δ LBR	-0.113	-0.061	-0.056	**-0.258**	**-0.244**	-0.119
	(0.546)	(0.712)	(0.713)	(0.054)	(0.079)	(0.324)
%Δ PCC	-0.083	-0.139	-0.163	-0.058	-0.120	-0.073
	(0.635)	(0.380)	(0.297)	(0.657)	(0.359)	(0.495)
SHORT	**-0.179**	**-0.277**	**-0.321**	-0.066	-0.054	**-0.176**
	(0.023)	(0.001)	(0.000)	(0.352)	(0.551)	(0.027)
TERM	**-0.464**	**-0.596**	**-0.623**	-0.141	-0.084	-0.178
	(0.002)	(0.000)	(0.000)	(0.371)	(0.658)	(0.294)
UINF	1.070	**1.290**	**1.385**	0.852	0.863	**1.143**
	(0.123)	(0.078)	(0.054)	(0.109)	(0.121)	(0.021)
LL	-1402.58	-1240.81	-1223.66	-1387.00	-1301.17	-1300.86
H_0:Exogenous=0	**54.496**	**83.615**	**98.458**	**116.027**	**82.640**	**68.065**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)

Table 3: Average partial effects (p-values in parenthesis) from the strictly exogenous specification, $h_{m+1} = Bx_m x_m' B'$. H_0:Exogenous tests the null that all of the non-constant parameters are zero (p-value in parenthesis). There is strong evidence that many of the variables lead the realized variances. There are also a number of interesting asymmetries, particularly across size for SHORT and TERM and across BE/ME for %Δ CLI. **Bold** denotes p-values below 10%.

Univariate Models - Expanded Variables

	SL	SM	SH	BL	BM	BH
CONST	**4.419**	**4.386**	**4.348**	**3.206**	**3.051**	**3.634**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
%Δ CLI	-0.225	-0.206	-0.182	**-0.345**	-0.236	**-0.397**
	(0.268)	(0.331)	(0.383)	(0.031)	(0.171)	(0.013)
CLI_{1YR}	**-0.192**	**-0.162**	-0.072	**-0.276**	**-0.254**	**-0.135**
	(0.000)	(0.002)	(0.164)	(0.000)	(0.000)	(0.004)
DEF	**1.663**	**2.164**	**1.778**	**2.872**	**3.165**	**2.317**
	(0.002)	(0.000)	(0.002)	(0.000)	(0.000)	(0.000)
ΔDEF	-0.900	-0.927	-1.038	-0.868	**-1.348**	**-1.491**
	(0.309)	(0.238)	(0.134)	(0.166)	(0.040)	(0.008)
DEF_{1YR}	-0.410	-0.407	-0.187	**-0.714**	-0.417	**-0.480**
	(0.290)	(0.241)	(0.567)	(0.019)	(0.168)	(0.092)
%Δ IP	0.024	-0.081	-0.159	0.158	0.141	0.025
	(0.887)	(0.667)	(0.395)	(0.371)	(0.500)	(0.876)
IP_{1YR}	0.037	0.067	0.060	0.091	**0.128**	0.091
	(0.574)	(0.388)	(0.477)	(0.105)	(0.045)	(0.148)
%Δ LBR	-0.031	-0.042	-0.116	-0.029	0.080	-0.013
	(0.837)	(0.777)	(0.478)	(0.806)	(0.475)	(0.918)
LBR_{1YR}	-0.111	-0.118	-0.099	**-0.180**	**-0.291**	**-0.237**
	(0.343)	(0.365)	(0.477)	(0.067)	(0.011)	(0.023)
%Δ PCC	-0.218	-0.289	-0.310	-0.191	**-0.239**	-0.214
	(0.177)	(0.107)	(0.110)	(0.138)	(0.080)	(0.120)
PCC_{1YR}	0.159	0.173	0.179	**0.149**	0.155	**0.164**
	(0.145)	(0.156)	(0.167)	(0.079)	(0.115)	(0.077)
SHORT	**-0.139**	**-0.312**	**-0.414**	-0.091	-0.073	**-0.257**
	(0.098)	(0.000)	(0.000)	(0.179)	(0.341)	(0.000)
TERM	**-0.591**	**-0.708**	**-0.638**	**-0.377**	**-0.390**	**-0.352**
	(0.002)	(0.000)	(0.001)	(0.027)	(0.041)	(0.038)
ΔTERM	**-0.653**	**-0.612**	-0.388	**-0.609**	**-0.732**	**-0.476**
	(0.033)	(0.031)	(0.112)	(0.008)	(0.004)	(0.035)
$TERM_{1YR}$	**0.342**	**0.350**	**0.362**	**0.193**	**0.139**	**0.243**
	(0.000)	(0.000)	(0.000)	(0.018)	(0.089)	(0.002)
UINF	**1.432**	**1.543**	**1.512**	**1.155**	**1.147**	**1.158**
	(0.006)	(0.005)	(0.006)	(0.010)	(0.011)	(0.008)
INF_{1YR}	**-0.230**	-0.099	0.079	**-0.230**	**-0.174**	0.029
	(0.034)	(0.285)	(0.403)	(0.006)	(0.030)	(0.669)
LL	-1392.573	-1233.014	-1214.721	-1375.368	-1288.082	-1293.221
H_0:Exogenous=0	(127.714)	(164.716)	(210.000)	(194.602)	(224.165)	(123.643)
	0.000	**0.000**	**0.000**	**0.000**	**0.000**	**0.000**

Table 4: Average partial effects (p-values in parenthesis) from the strictly exogenous specification, $h_{m+1} = Bx_m x_m' B'$ using the expanded of exogenous variables. H_0:Exogenous tests the null that all of the non-constant parameters are zero (p-value in parenthesis). A large number of the transformed variables are significant, especially those which measure year over year changes. **Bold** denotes p-values below 10%.

Univariate Models - Core Variables with ARCH

	SL	SM	SH	BL	BM	BH
CONST	**0.954**	**1.465**	**1.640**	**0.435**	**0.444**	**1.347**
	(0.005)	(0.000)	(0.000)	(0.024)	(0.054)	(0.000)
%Δ CLI	0.088	0.019	0.032	-0.153	-0.002	-0.130
	(0.519)	(0.879)	(0.816)	(0.121)	(0.984)	(0.246)
DEF	-0.063	0.344	0.518	0.255	0.191	**0.547**
	(0.842)	(0.281)	(0.104)	(0.275)	(0.466)	(0.042)
SHORT	0.073	-0.028	-0.104	0.053	0.054	-0.055
	(0.364)	(0.721)	(0.168)	(0.235)	(0.267)	(0.344)
TERM	-0.021	-0.184	**-0.267**	0.058	0.035	-0.054
	(0.901)	(0.257)	(0.057)	(0.472)	(0.671)	(0.692)
UINF	**1.186**	**1.140**	0.745	0.476	**0.570**	0.658
	(0.025)	(0.058)	(0.128)	(0.104)	(0.064)	(0.120)
ARCH	**0.834**	**0.787**	**0.867**	**0.883**	**0.915**	**0.711**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
LL	-1370.57	-1220.55	-1205.09	-1349.10	-1266.46	-1286.12
H_0:Exogenous=0	**14.178**	**18.198**	**18.069**	**34.653**	**22.151**	**12.720**
	(0.015)	(0.003)	(0.003)	(0.000)	(0.000)	(0.026)
ARCH (No Exog.)	**0.844**	**0.838**	**0.955**	**1.007**	**1.015**	**0.795**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
LL (No Exog.)	-1373.42	-1223.25	-1207.41	-1351.26	-1268.13	-1287.19

Table 5: Average partial effects (p-values in parenthesis) from a model which includes both exogenous variables and ARCH effects, $h_{m+1} = Bx_m x_m' B' + aRV_m$. H_0:Exogenous tests the null that all of the parameters on the exogenous variables are zero (p-value in parenthesis). While including ARCH effects weakens the significance of many of the variables, most notably the highly persistent variables, there is still strong evidence of significance of the exogenous variables. The last two lines are the ARCH parameter and log-likelihood from a model without exogenous variables ($h_{m+1} = b_1^2 + aRV_m$). **Bold** denotes p-values below 10%.

Univariate Models - Expanded Variables with ARCH

	SL	SM	SH	BL	BM	BH
CONST	**1.224**	**1.842**	**1.970**	**0.592**	**0.816**	**1.627**
	(0.006)	(0.000)	(0.001)	(0.021)	(0.009)	(0.000)
%Δ CLI	0.003	-0.062	-0.018	-0.171	-0.004	-0.186
	(0.980)	(0.632)	(0.897)	(0.130)	(0.971)	(0.164)
CLI_{1YR}	-0.016	-0.027	-0.013	-0.027	**-0.068**	-0.034
	(0.683)	(0.530)	(0.748)	(0.500)	(0.060)	(0.392)
DEF	-0.236	0.158	0.188	0.350	0.559	0.544
	(0.658)	(0.751)	(0.708)	(0.348)	(0.103)	(0.108)
ΔDEF	-0.637	-0.577	-0.366	-0.221	-0.544	**-0.878**
	(0.331)	(0.304)	(0.444)	(0.707)	(0.288)	(0.041)
DEF_{1YR}	0.515	0.394	0.309	0.132	0.011	0.076
	(0.197)	(0.325)	(0.407)	(0.701)	(0.971)	(0.824)
SHORT	0.111	0.002	-0.086	0.085	0.081	-0.027
	(0.285)	(0.984)	(0.373)	(0.164)	(0.201)	(0.719)
TERM	0.037	-0.159	-0.229	0.054	-0.010	-0.074
	(0.868)	(0.478)	(0.259)	(0.650)	(0.934)	(0.684)
ΔTERM	-0.320	**-0.385**	**-0.423**	-0.157	-0.253	-0.269
	(0.143)	(0.094)	(0.071)	(0.382)	(0.207)	(0.175)
$TERM_{1YR}$	0.119	**0.173**	**0.189**	0.056	0.063	**0.140**
	(0.205)	(0.070)	(0.086)	(0.425)	(0.421)	(0.061)
UINF	**1.424**	**1.478**	**1.062**	**0.630**	**0.923**	**0.918**
	(0.012)	(0.025)	(0.056)	(0.054)	(0.008)	(0.048)
INF_{1YR}	-0.038	-0.032	0.029	**-0.076**	**-0.135**	-0.054
	(0.487)	(0.563)	(0.606)	(0.100)	(0.005)	(0.208)
ARCH	**0.764**	**0.699**	**0.757**	**0.860**	**0.849**	**0.658**
	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)	(0.000)
LL	-1368.83	-1218.43	-1202.94	-1348.46	-1264.60	-1284.97
H_0:Exogenous=0	**28.559**	**35.285**	**36.477**	**47.139**	**45.573**	**28.314**
	(0.003)	(0.000)	(0.000)	(0.000)	(0.000)	(0.003)

Table 6: Average partial effects (p-values in parenthesis) from a model which includes both exogenous variables and ARCH effects, $h_{m+1} = Bx_m x_m' B' + aRV_m$. H_0:Exogenous tests the null that all of the parameters on the explanatory variables are zero (p-value in parenthesis). Variables which were insignificant in the strictly exogenous formulation were excluded when ARCH effects were introduced. Including ARCH effects reduces the significance of many of the variables, most notably the highly persistent variables. However, there is still strong evidence of significance of exogenous variables in all of the series. **Bold** denotes p-values below 10%.

Variance Fit of Univariate Models

	SL	SM	SH	BL	BM	BH
Core	0.020	0.026	0.027	0.025	0.019	0.038
Core and ARCH	0.089	0.047	0.028	0.079	0.042	0.044
ARCH	0.088	0.045	0.026	0.077	0.041	0.040
Expanded	0.067	0.055	0.051	0.052	0.034	0.065
Expanded and ARCH	0.097	0.058	0.036	0.083	0.045	0.053

Variance Fit of Univariate Models (October 1987 dummy)

	SL	SM	SH	BL	BM	BH
Core	0.482	0.606	0.673	0.668	0.760	0.728
Core and ARCH	0.549	0.623	0.667	0.718	0.782	0.734
ARCH	0.549	0.621	0.664	0.717	0.782	0.731
Expanded	0.521	0.628	0.690	0.690	0.771	0.748
Expanded and ARCH	0.556	0.632	0.674	0.721	0.785	0.741

Table 7: R^2's from a regression of realized variances on the fit variance from the five models considered. The top panel includes the full sample while the bottom excludes October 1987. The regression fit is $RV_m = \alpha + \beta h_m (+\gamma D_{OCT97}) + \epsilon_m$. The highest R^2 are produced by the augmented ARCH model with the expanded set of variables.

Multivariate Model - Joint Tests of Significance

	(1)	(2)		(3)	(4)
H_0 :Exogenous=0	**1268.8**	**592.6**		**5936.7**	**2237.6**
	(0.000)	(0.000)		(0.000)	(0.000)
d.f.	105	105		231	231

	(1)	(2)		(3)	(4)
%ΔCLI	**89.885**	**42.676**	%ΔCLI	**71.997**	**43.389**
	(0.000)	(0.001)		(0.000)	(0.003)
DEF	**125.645**	**55.338**	CLI_{1YR}	**160.114**	**48.860**
	(0.000)	(0.039)		(0.000)	(0.001)
SHORT	**315.839**	**131.664**	DEF	**101.740**	**44.020**
	(0.000)	(0.000)		(0.000)	(0.002)
TERM	**336.303**	**154.86**	ΔDEF	**49.931**	**39.775**
	(0.000)	(0.000)		(0.000)	(0.008)
UINF	**30.443**	18.961	DEF_{1YR}	**49.422**	19.113
	(0.083)	(0.825)		(0.000)	(0.578)
			SHORT	**300.082**	**154.105**
				(0.000)	(0.000)
			TERM	**210.745**	**120.918**
				(0.000)	(0.000)
			TERM_{1YR}	24.619	**31.822**
				(0.264)	(0.061)
			ΔTERM	**95.254**	**77.368**
				(0.000)	(0.000)
			UINF	**30.580**	21.061
				(0.081)	(0.455)
			INF_{1YR}	**234.358**	**140.401**
				(0.000)	(0.000)

Table 8: Test statistics (p-values in parenthesis). Each set of test statistics represents a model with only exogenous variables (columns (1) and (3)) and with both exogenous variables and ARCH effects (columns (2) and (4)). The top panel contains test statistics for joint significance of all parameters on the exogenous variables and is distributed $\chi^2_{NK(K+1)/2}$. The bottom panel tests the null that all parameters on a particular exogenous variable had no effect. $H_0 : b_{ijn}=0 \quad \forall \; i,j = 1,\ldots,K, \; i \geq j$. The distribution of each test statistic under the null that all of the coefficients are zero is a $\chi^2_{K(K+1)/2}$. When $K = 6$, the test has have 21 d.f. **Bold** denotes p-values below 10%

ARCH

	SL	SM	SB	HL	HM	HB
SL	0.615	0.585	0.567	0.631	0.606	0.594
SM	-	0.577	0.563	0.605	0.594	0.577
SB	-	-	0.561	0.585	0.577	0.565
HL	-	-	-	0.747	0.720	0.695
HM	-	-	-	-	0.723	0.704
HB	-	-	-	-	-	0.693

Core Variables with ARCH

	SL	SM	SB	HL	HM	HB
SL	0.600	0.565	0.546	0.573	0.549	0.533
SM	-	0.544	0.526	0.540	0.526	0.509
SB	-	-	0.517	0.519	0.508	0.495
HL	-	-	-	0.665	0.638	0.607
HM	-	-	-	-	0.645	0.621
HB	-	-	-	-	-	0.604

Expanded Variables with ARCH

	SL	SM	SB	HL	HM	HB
SL	0.518	0.486	0.465	0.518	0.489	0.467
SM	-	0.467	0.451	0.488	0.470	0.447
SB	-	-	0.442	0.468	0.453	0.432
HL	-	-	-	0.611	0.581	0.545
HM	-	-	-	-	0.582	0.553
HB	-	-	-	-	-	0.531

Table 9: Parameters on lagged realized covariance in the three multivariate specifications which included ARCH effects. The model for the conditional covariance in all three of these specifications can be described by $H_{m+1} = B(I_k \otimes x_m x_m')B' + AA' \odot RV_m$. The parameters shown are AA', not A. The difference between the three models is determined by the choice of x_m. In the top panel, x_m includes only a constant, while in the second and third panels x_m will correspond to one of the two sets of explanatory variables (core set in the 2^{nd} panel, expanded in the 3^{rd}.)

Variance Results - Multivariate Model

	SL	SM	SH	BL	BM	BH
CONST	**19.206**	**14.043**	**13.831**	**13.283**	**10.109**	**11.881**
%Δ CLI	**-2.056**	**-1.495**	**-1.239**	**-3.071**	**-2.077**	**-2.299**
DEF	**4.193**	**4.743**	**4.926**	**5.403**	**5.251**	**5.533**
SHORT	**-0.839**	**-0.907**	**-0.982**	**-0.348**	**-0.283**	**-0.601**
TERM	**-2.173**	**-1.837**	**-1.799**	**-0.495**	**-0.258**	**-0.443**
UINF	**3.073**	2.815	3.038	2.729	**2.762**	3.437

Variance Results - Multivariate Model with ARCH

	SL	SM	SH	BL	BM	BH
CONST	**6.107**	**6.117**	**7.000**	**2.314**	**2.675**	**4.684**
%Δ CLI	**-0.236**	**-0.509**	**-0.532**	**-0.655**	**-0.374**	**-0.676**
DEF	**0.455**	**1.558**	**2.133**	**0.710**	**1.209**	**2.052**
SHORT	**0.100**	**-0.216**	**-0.408**	**0.358**	**0.184**	**-0.125**
TERM	**-0.304**	**-0.646**	**-0.886**	**0.559**	**0.306**	**-0.067**
UINF	2.960	2.415	2.305	1.808	1.821	2.313

Variance Results - Multivariate Model with ARCH

	SL	SM	SH	BL	BM	BH
CONST	**19.556**	**14.284**	**13.641**	**13.749**	**10.292**	**11.864**
%Δ CLI	**-0.909**	**-0.950**	**-1.040**	**-1.580**	**-1.203**	**-1.934**
CLI_{1YR}	**-0.801**	**-0.471**	**-0.219**	**-1.003**	**-0.790**	**-0.469**
DEF	**5.541**	**5.267**	**4.153**	**9.510**	**8.941**	**7.399**
ΔDEF	-4.551	-3.352	**-3.001**	-3.663	**-4.905**	**-5.054**
DEF_{1YR}	-1.142	**-0.789**	-0.234	**-2.873**	**-1.362**	-1.506
SHORT	**-0.401**	**-0.791**	**-1.062**	**-0.036**	**0.057**	**-0.648**
TERM	-1.955	**-1.680**	**-1.523**	**-0.706**	**-0.441**	**-0.617**
ΔTERM	**-2.979**	-2.102	-1.662	**-2.467**	**-2.343**	**-1.867**
$TERM_{1YR}$	**0.939**	**0.583**	**0.725**	**0.320**	**0.159**	**0.576**
UINF	**6.737**	**5.261**	4.963	**5.439**	**5.233**	**5.341**
INF_{1YR}	**-0.831**	**-0.236**	**0.321**	**-1.131**	**-1.064**	**-0.190**

Variance Results - Multivariate Model with ARCH

	SL	SM	SH	BL	BM	BH
CONST	**7.749**	**7.243**	**7.688**	**3.620**	**4.085**	**5.804**
%Δ CLI	**-0.329**	**-0.533**	**-0.592**	**-0.535**	**-0.279**	**-0.803**
CLI_{1YR}	**-0.239**	**-0.197**	**-0.109**	**-0.328**	**-0.347**	**-0.194**
DEF	**0.765**	**1.772**	**1.692**	**2.736**	**3.556**	**3.156**
ΔDEF	-3.201	-2.232	-2.088	-0.571	-2.054	-2.415
DEF_{1YR}	0.844	0.353	0.447	-0.486	-0.483	-0.184
SHORT	**0.183**	**-0.168**	**-0.426**	**0.354**	**0.218**	**-0.145**
TERM	-0.266	**-0.648**	**-0.772**	**0.452**	**0.091**	**-0.155**
ΔTERM	**-1.448**	**-1.331**	**-1.422**	**-0.952**	-1.167	**-1.527**
$TERM_{1YR}$	**0.573**	**0.478**	**0.565**	0.051	**0.067**	**0.384**
UINF	**4.758**	**3.925**	3.647	3.037	3.484	3.747
INF_{1YR}	**-0.238**	**-0.147**	**0.128**	**-0.427**	**-0.567**	**-0.210**

Table 10: This table contains the average partial effects of a change in one of the exogenous variables with respect to the annualized volatility (equation 30). The top panel contains results from the multivariate specification with only the core set of explanatory variables. The second contains results from the model with the core explanatory variables when the model had ARCH effects. The third and fourth contain the APEs from the multivariate model using the expanded set of explanatory variables without and with ARCH effects, respectively. **Bold** denotes that the effect of the exogenous variable is significant at the 10% level.

Multivariate Model - Correlation Results, Core Model

%Δ CLI

	SM	SH	BL	BM	BH
SL	**-0.000**	**-0.001**	**-0.021**	**-0.023**	**-0.032**
SM		**-0.025**	**-0.036**	**-0.031**	**-0.028**
SH			**-0.007**	**-0.034**	**-0.005**
BL				**-0.025**	**-0.026**
BM					**-0.027**

DEF

	SM	SH	BL	BM	BH
SL	**0.006**	**0.003**	**0.156**	**0.087**	**0.111**
SM		**0.100**	**0.181**	**0.124**	**0.130**
SH			**0.009**	**0.140**	**0.035**
BL				**0.080**	**0.069**
BM					**0.030**

SHORT

	SM	SH	BL	BM	BH
SL	**0.002**	**0.002**	**-0.011**	**-0.013**	**-0.017**
SM		**-0.011**	**-0.018**	**-0.017**	**-0.022**
SH			**-0.000**	**-0.021**	0.013
BL				**-0.011**	0.009
BM					0.005

TERM

	SM	SH	BL	BM	BH
SL	**0.001**	**-0.002**	**-0.031**	**-0.021**	**-0.028**
SM		**-0.027**	**-0.030**	**-0.031**	**-0.040**
SH			**-0.004**	**-0.039**	**0.027**
BL				**-0.014**	**0.031**
BM					**0.020**

UINF

	SM	SH	BL	BM	BH
SL	**0.028**	**0.050**	**0.008**	**-0.015**	**0.029**
SM		**-0.024**	**0.047**	**0.002**	**0.012**
SH			**0.027**	**0.017**	**0.029**
BL				**0.026**	**0.091**
BM					**0.081**

Table 11: Average partial effects on correlation from a multivariate model using only the core set of explanatory variables. **Bold** indicates the p-value is below 10%, where the p-value is from a test whether the variable appears in the conditional covariance between two portfolios. The strongest effects are seen between the small and large firm portfolios.

Multivariate Model - Correlation Results, Core Model with ARCH

%Δ CLI

	SM	SH	BL	BM	BH
SL	0.002	-0.001	-0.033	**-0.044**	**-0.041**
SM		**-0.038**	**-0.048**	**-0.040**	**-0.041**
SH			-0.005	**-0.045**	-0.003
BL				**-0.046**	**-0.024**
BM					-0.006

DEF

	SM	SH	BL	BM	BH
SL	-0.005	-0.006	0.090	**0.096**	**0.113**
SM		0.077	**0.111**	**0.109**	**0.128**
SH			0.006	**0.125**	0.002
BL				**0.091**	0.026
BM					0.009

SHORT

	SM	SH	BL	BM	BH
SL	0.004	0.004	-0.009	**-0.012**	**-0.016**
SM		-0.007	**-0.018**	-0.015	**-0.022**
SH			0.000	**-0.022**	0.010
BL				**-0.010**	0.002
BM					**0.006**

TERM

	SM	SH	BL	BM	BH
SL	**0.000**	**-0.003**	**-0.017**	**-0.006**	**-0.015**
SM		-0.003	-0.031	**-0.018**	**-0.030**
SH			**-0.003**	**-0.033**	0.013
BL				**-0.001**	**0.004**
BM					**0.011**

UINF

	SM	SH	BL	BM	BH
SL	**0.024**	**0.041**	**0.059**	**0.015**	**0.068**
SM		**0.026**	**0.092**	**0.048**	**0.070**
SH			**0.026**	**0.072**	**0.011**
BL				**0.039**	**0.044**
BM					**0.052**

Table 12: Average partial effects on correlation from a multivariate model using the core set of explanatory variables augmented with ARCH. **Bold** indicates the p-value is below 10%, where the p-value is from a test whether the variable appears in the conditional covariance between two portfolios. While the effects are generally reduced from the strictly exogenous model, there is widespread evidence of significance. The strongest effects are seen between the small and large firm portfolios.

Multivariate Model - Correlation Results, Expanded Model

%Δ CLI

	SM	SH	BL	BM	BH
SL	**0.003**	**0.002**	**-0.019**	**-0.016**	**-0.030**
SM		**-0.014**	**-0.053**	**-0.026**	**-0.041**
SH			**-0.005**	**-0.044**	**-0.002**
BL				**-0.020**	**-0.027**
BM					**-0.017**

CLI_{1YR}

	SM	SH	BL	BM	BH
SL	**-0.001**	**-0.001**	**-0.001**	**-0.003**	**-0.000**
SM		**-0.004**	**0.007**	**-0.002**	**0.006**
SH			**-0.001**	**0.005**	**-0.003**
BL				**-0.001**	**-0.002**
BM					**-0.006**

DEF

	SM	SH	BL	BM	BH
SL	**0.000**	**-0.020**	**0.078**	**0.072**	**0.113**
SM		**0.060**	**0.049**	**0.116**	**0.074**
SH			**0.008**	**0.076**	**0.052**
BL				**0.058**	**0.053**
BM					**0.070**

ΔDEF

	SM	SH	BL	BM	BH
SL	**0.002**	**0.018**	**-0.017**	**-0.055**	**-0.111**
SM		**-0.020**	**0.034**	**-0.073**	**-0.041**
SH			**-0.007**	**-0.014**	**0.001**
BL				**-0.064**	**-0.011**
BM					**-0.021**

DEF_{1YR}

	SM	SH	BL	BM	BH
SL	**-0.001**	0.010	0.022	**0.011**	**-0.017**
SM		**0.021**	0.051	**-0.022**	0.016
SH			0.005	0.018	**0.016**
BL				**0.021**	**0.008**
BM					**-0.023**

SHORT

	SM	SH	BL	BM	BH
SL	**-0.000**	**-0.002**	**-0.027**	**-0.014**	**-0.032**
SM		-0.013	**-0.040**	**-0.033**	**-0.038**
SH			**-0.002**	**-0.040**	**0.014**
BL				**-0.012**	**-0.001**
BM					**-0.002**

TERM

	SM	SH	BL	BM	BH
SL	**0.005**	**0.007**	**-0.004**	**-0.010**	**-0.020**
SM		**-0.005**	**0.002**	**-0.021**	**-0.026**
SH			**0.001**	**-0.020**	0.035
BL				**-0.003**	**0.043**
BM					**0.024**

ΔTERM

	SM	SH	BL	BM	BH
SL	**-0.012**	**-0.009**	**-0.018**	**-0.015**	**-0.008**
SM		**-0.005**	**-0.012**	**-0.014**	**-0.009**
SH			**-0.008**	**-0.016**	**-0.032**
BL				**-0.004**	**-0.028**
BM					**-0.017**

$TERM_{1YR}$

	SM	SH	BL	BM	BH
SL	**-0.004**	**-0.008**	-0.022	**-0.004**	**-0.014**
SM		**-0.009**	**-0.022**	**-0.017**	**-0.003**
SH			**-0.003**	**-0.011**	-0.011
BL				**-0.005**	-0.020
BM					-0.017

UINF

	SM	SH	BL	BM	BH
SL	**0.037**	**0.062**	**0.064**	**0.030**	**0.073**
SM		**0.024**	**0.096**	**0.052**	**0.056**
SH			**0.036**	**0.065**	**0.053**
BL				**0.065**	**0.104**
BM					**0.093**

INF_{1YR}

	SM	SH	BL	BM	BH
SL	0.004	0.011	**0.033**	**0.005**	**0.019**
SM		**0.010**	**0.057**	**0.021**	**0.036**
SH			0.003	**0.040**	**0.001**
BL				**0.008**	**0.021**
BM					**0.003**

Table 13: Average partial effects on correlation from a multivariate model using only the expanded set of explanatory variables. **Bold** indicates the p-value is below 10%, where the p-value is from a test whether the variable appears in the conditional covariance between two portfolios. The strongest effects are seen between the small and large firm portfolios.

Multivariate Model - Correlation Results, Expanded Model with ARCH

%Δ CLI

	SM	SH	BL	BM	BH
SL	0.003	0.002	-0.017	**-0.037**	**-0.034**
SM		-0.027	**-0.034**	**-0.031**	**-0.038**
SH			-0.005	**-0.038**	0.001
BL				**-0.040**	**-0.017**
BM					**-0.003**

CLI_{1YR}

	SM	SH	BL	BM	BH
SL	**-0.001**	**-0.002**	**-0.005**	**-0.003**	**-0.004**
SM		**-0.004**	**-0.002**	**-0.004**	0.001
SH			**-0.001**	**-0.000**	**-0.004**
BL				**-0.003**	**-0.005**
BM					**-0.004**

DEF

	SM	SH	BL	BM	BH
SL	-0.002	-0.005	**0.106**	**0.086**	**0.132**
SM		**0.074**	**0.099**	**0.127**	**0.107**
SH			0.007	**0.102**	**0.029**
BL				**0.081**	**0.060**
BM					**0.051**

ΔDEF

	SM	SH	BL	BM	BH
SL	**-0.005**	**0.001**	**-0.061**	-0.063	**-0.108**
SM		**-0.032**	**0.019**	**-0.095**	**-0.019**
SH			**-0.010**	**-0.014**	-0.018
BL				-0.092	**-0.045**
BM					**0.009**

DEF_{1YR}

	SM	SH	BL	BM	BH
SL	0.001	-0.003	-0.028	0.001	-0.027
SM		-0.014	-0.033	-0.026	-0.021
SH			0.002	-0.017	0.011
BL				0.003	-0.008
BM					-0.008

SHORT

	SM	SH	BL	BM	BH
SL	**0.003**	0.002	-0.010	**-0.005**	**-0.014**
SM		-0.004	**-0.026**	-0.015	**-0.026**
SH			-0.001	**-0.026**	0.012
BL				**-0.003**	**-0.001**
BM					**0.004**

TERM

	SM	SH	BL	BM	BH
SL	**0.003**	**0.001**	-0.016	**-0.006**	**-0.019**
SM		-0.003	-0.024	-0.021	**-0.029**
SH			**-0.000**	**-0.028**	0.017
BL				**-0.001**	**0.008**
BM					**0.013**

ΔTERM

	SM	SH	BL	BM	BH
SL	**-0.005**	**0.006**	**-0.008**	**0.014**	**-0.004**
SM		**0.003**	**-0.029**	**-0.005**	**-0.024**
SH			**-0.006**	**-0.032**	**-0.004**
BL				**0.024**	**-0.021**
BM					**-0.031**

$TERM_{1YR}$

	SM	SH	BL	BM	BH
SL	**-0.001**	**-0.003**	-0.005	**0.007**	**0.002**
SM		0.004	**0.001**	**-0.000**	**0.012**
SH			**-0.000**	**0.009**	-0.007
BL				**0.005**	-0.009
BM					-0.009

UINF

	SM	SH	BL	BM	BH
SL	**0.031**	**0.045**	**0.074**	**0.042**	**0.090**
SM		**0.045**	**0.099**	**0.069**	**0.084**
SH			**0.032**	**0.084**	**0.030**
BL				**0.066**	**0.057**
BM					**0.067**

INF_{1YR}

	SM	SH	BL	BM	BH
SL	0.001	0.004	**0.003**	**-0.006**	**-0.004**
SM		**-0.003**	**0.019**	**-0.002**	**0.013**
SH			0.002	0.013	**-0.005**
BL				-0.007	0.002
BM					**-0.004**

Table 14: Average partial effects on correlation from a multivariate model using the expanded set of explanatory variables augmented with ARCH. **Bold** indicates the p-value is below 10%, where the p-value is from a test whether the variable appears in the conditional covariance between two portfolios. The effects are generally reduced from the strictly exogenous model although there is widespread evidence of significance. The strongest effects are seen between the small and large firm portfolios.

Bivariate Models - Correlation Results, Core Model with ARCH

%Δ CLI

	SM	SH	BL	BM	BH
SL	0.005	0.006	-0.011	-0.030	-0.009
SM		**-0.018**	**-0.023**	-0.013	-0.009
SH			**0.003**	-0.018	**-0.004**
BL				-0.032	-0.024
BM					-0.001

DEF

	SM	SH	BL	BM	BH
SL	**-0.006**	**-0.015**	0.028	**0.067**	**0.032**
SM		0.029	**0.064**	0.047	**0.083**
SH			**0.012**	**0.080**	0.020
BL				**0.059**	**0.037**
BM					**0.008**

SHORT

	SM	SH	BL	BM	BH
SL	**0.004**	**0.004**	-0.001	**-0.009**	**-0.004**
SM		-0.001	**-0.012**	**-0.004**	**-0.015**
SH			**0.001**	**-0.015**	0.005
BL				**-0.010**	**-0.001**
BM					**0.005**

TERM

	SM	SH	BL	BM	BH
SL	**0.001**	**-0.000**	**-0.004**	**0.001**	**0.003**
SM		**0.004**	-0.017	**-0.002**	**-0.018**
SH			**-0.001**	**-0.020**	0.002
BL				**0.006**	**-0.000**
BM					**0.008**

UINF

	SM	SH	BL	BM	BH
SL	**0.033**	0.045	**0.091**	0.029	0.082
SM		0.043	**0.126**	0.074	0.067
SH			0.026	0.085	0.009
BL				0.045	**0.039**
BM					0.055

Table 15: Average partial effects on correlation from a the set of all bivariate models using the core set of explanatory variables augmented with ARCH. **Bold** indicates the p-value is below 10%, where the p-value is from a test whether the variable appears in the conditional covariance between two portfolios. The effects are similar to the full multivariate specification both in magnitude and in significance.

Bivariate Models - Correlation Results, Expanded Model with ARCH

%Δ CLI

	SM	SH	BL	BM	BH
SL	**0.001**	0.008	0.012	**-0.031**	**-0.018**
SM		-0.020	-0.017	**-0.021**	-0.014
SH			-0.001	-0.020	-0.005
BL				-0.034	-0.014
BM					-0.003

CLI_{1YR}

	SM	SH	BL	BM	BH
SL	-0.001	-0.003	**-0.008**	**-0.001**	**-0.003**
SM		0.000	0.000	**-0.003**	**0.002**
SH			**-0.001**	**0.001**	**-0.002**
BL				-0.002	**-0.008**
BM					**-0.004**

DEF

	SM	SH	BL	BM	BH
SL	**0.002**	**0.006**	**0.103**	0.080	0.084
SM		0.039	**0.060**	0.088	0.075
SH			0.019	0.079	**0.028**
BL				0.080	**0.076**
BM					0.070

ΔDEF

	SM	SH	BL	BM	BH
SL	-0.019	0.010	0.021	-0.024	-0.103
SM		0.003	0.068	-0.074	0.035
SH			-0.000	0.031	-0.029
BL				-0.069	-0.068
BM					-0.022

DEF_{1YR}

	SM	SH	BL	BM	BH
SL	**0.003**	-0.006	**-0.044**	0.017	-0.021
SM		0.022	0.009	-0.005	0.003
SH			0.003	0.013	0.020
BL				0.014	-0.014
BM					-0.009

SHORT

	SM	SH	BL	BM	BH
SL	**-0.000**	**0.001**	0.003	**-0.002**	**-0.008**
SM		0.002	**-0.017**	**-0.008**	**-0.019**
SH			**-0.002**	**-0.018**	**0.010**
BL				**-0.002**	**0.007**
BM					**0.014**

TERM

	SM	SH	BL	BM	BH
SL	**-0.001**	**-0.000**	**-0.024**	**0.001**	**-0.010**
SM		**0.002**	-0.018	**-0.010**	**-0.023**
SH			**-0.002**	**-0.021**	0.006
BL				**0.003**	**-0.009**
BM					**0.010**

ΔTERM

	SM	SH	BL	BM	BH
SL	0.004	0.038	-0.009	0.035	-0.008
SM		0.027	0.069	0.005	0.030
SH			0.019	0.041	0.019
BL				0.025	**0.044**
BM					-0.012

$TERM_{1YR}$

	SM	SH	BL	BM	BH
SL	**0.005**	**0.001**	**-0.010**	**0.008**	0.013
SM		**0.001**	-0.011	**0.007**	0.011
SH			**0.003**	0.004	**-0.007**
BL				0.009	**-0.014**
BM					**0.004**

UINF

	SM	SH	BL	BM	BH
SL	**0.039**	**0.047**	**0.084**	**0.039**	**0.100**
SM		**0.036**	**0.109**	**0.083**	0.064
SH			**0.034**	**0.072**	**0.022**
BL				**0.072**	**0.078**
BM					**0.064**

INF_{1YR}

	SM	SH	BL	BM	BH
SL	**0.005**	**-0.001**	**-0.014**	-0.011	**-0.005**
SM		-0.006	**0.009**	**-0.005**	**0.007**
SH			**-0.001**	**0.004**	**-0.007**
BL				**-0.013**	**-0.005**
BM					**-0.012**

Table 16: Average partial effects on correlation from a the set of all bivariate models using the expanded set of explanatory variables augmented with ARCH. **Bold** indicates the p-value is below 10%, where the p-value is from a test whether the variable appears in the conditional covariance between two portfolios. The effects are similar to the full multivariate specification both in magnitude and in significance.

Variance and Correlation Fit of Multivariate Models

ARCH

	SL	SM	SH	BL	BM	BH
SL	0.088	0.057	0.098	0.065	0.181	0.298
SM	-	0.045	0.076	0.041	0.137	0.224
SH	-	-	0.026	0.078	0.116	0.199
BL	-	-	-	0.077	0.144	0.263
BM	-	-	-	-	0.041	0.212
BH	-	-	-	-	-	0.040

Exogenous - Core

	SL	SM	SH	BL	BM	BH
SL	0.019	0.043	0.018	0.131	0.142	0.095
SM	-	0.027	0.032	0.093	0.091	0.081
SH	-	-	0.028	0.049	0.070	0.065
BL	-	-	-	0.026	0.155	0.050
BM	-	-	-	-	0.020	0.109
BH	-	-	-	-	-	0.037

Exogenous - Core and ARCH

	SL	SM	SH	BL	BM	BH
SL	0.090	0.089	0.117	0.106	0.229	0.329
SM	-	0.050	0.092	0.070	0.173	0.254
SH	-	-	0.031	0.095	0.143	0.205
BL	-	-	-	0.080	0.200	0.286
BM	-	-	-	-	0.044	0.267
BH	-	-	-	-	-	0.045

Exogenous - Expanded

	SL	SM	SH	BL	BM	BH
SL	0.062	0.062	0.098	0.152	0.197	0.268
SM	-	0.053	0.078	0.112	0.154	0.243
SH	-	-	0.050	0.088	0.138	0.212
BL	-	-	-	0.048	0.140	0.106
BM	-	-	-	-	0.032	0.166
BH	-	-	-	-	-	0.054

Exogenous - Expanded and ARCH

	SL	SM	SH	BL	BM	BH
SL	0.102	0.111	0.147	0.102	0.258	0.380
SM	-	0.062	0.115	0.065	0.195	0.309
SH	-	-	0.042	0.086	0.156	0.266
BL	-	-	-	0.087	0.202	0.293
BM	-	-	-	-	0.050	0.272
BH	-	-	-	-	-	0.055

Table 17: These 5 panels contain R^2's from a regression of realized variances (diagonal elements) or realized correlations (off-diagonal elements) on the fit variance or fit correlation from the five models considered in this paper. The regression fit is $RV_m^{ii} = \alpha + \beta H_m^{ii} + \epsilon_m$ where RC_m^{ij} and fit correlation are used in the off-diagonal elements.