Financial Econometrics
HT Week 4 Assignment Answers

February 2021

Exercise 7.2

Suppose $r_t = \sigma_t \varepsilon_t$ where $\sigma_t^2 = \omega + \alpha r_{t-1}^2 + \beta \sigma_{t-1}^2$, and $\varepsilon_t \overset{\text{i.i.d.}}{\sim} N(0,1)$. What conditions are required on the parameters ω, α, and β for r_t to be covariance stationary?

1. The variance must have finite memory, which requires $\alpha + \beta < 1$.
2. The variance must be non-negative, so that $\alpha \geq 0$ and $\beta \geq 0$.
3. The variance must stay away from zero, so $\omega > 0$

Exercise 7.5

Outline the steps the in Mincer-Zarnowitz framework to objectively evaluate a sequence of variance forecasts $\left\{\hat{\sigma}^2_{t+1|t}\right\}$.

Generalized Mincer-Zarnowitz regressions can be used to assess forecast optimality,

$$r^2_{t+h} - \hat{\sigma}^2_{t+h|t} = \gamma_0 + \gamma_1 \hat{\sigma}^2_{t+h|t} + \gamma_2 z_{1t} + \ldots + \gamma_{K+1} z_{Kt} + \eta_t$$

where z_{jt} are any instruments known at time t. Common choices for z_{jt} include r^2_t, $|r_t|$, r_t or indicator variables for the sign of the lagged return. The GMZ regression has a heteroskedastic variance and that a better estimator, GMZ-GLS, can be constructed as

$$\frac{r^2_{t+h} - \hat{\sigma}^2_{t+h|t}}{\hat{\sigma}^2_{t+h|t}} = \gamma_0 \frac{1}{\hat{\sigma}^2_{t+h|t}} + \gamma_1 1 + \gamma_2 \frac{z_{1t}}{\hat{\sigma}^2_{t+h|t}} + \ldots + \gamma_{K+1} \frac{z_{Kt}}{\hat{\sigma}^2_{t+h|t}} + \nu_t$$

$$\frac{r^2_{t+h}}{\hat{\sigma}^2_{t+h|t}} - 1 = \gamma_0 \frac{1}{\hat{\sigma}^2_{t+h|t}} + \gamma_1 1 + \gamma_2 \frac{z_{1t}}{\hat{\sigma}^2_{t+h|t}} + \ldots + \gamma_{K+1} \frac{z_{Kt}}{\hat{\sigma}^2_{t+h|t}} + \nu_t$$

by dividing both sized by the time t forecast, $\hat{\sigma}^2_{t+h|t}$ where $\nu_t = \eta_t / \hat{\sigma}^2_{t+h|t}$.
These models are estimated by OLS (or GLS) and the coefficients are tested under the null $H_0 : \gamma = 0$ against an alternative that one or more is non-zero. The test can be implemented as a Wald, LM or LR test.